BASIC CONTROL SYSTEMS

04 DATA DRIVEN PROCESS CONTROL

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WHERE STUDENTS MATTER

DATA DRIVEN METHOD

We were playing with the transfer function.

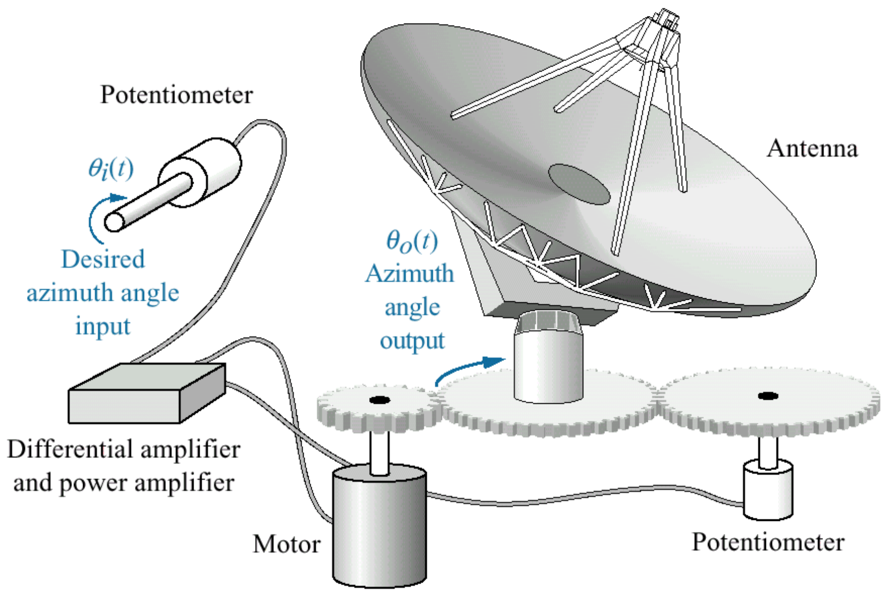
What if this is our system:

Blackbox

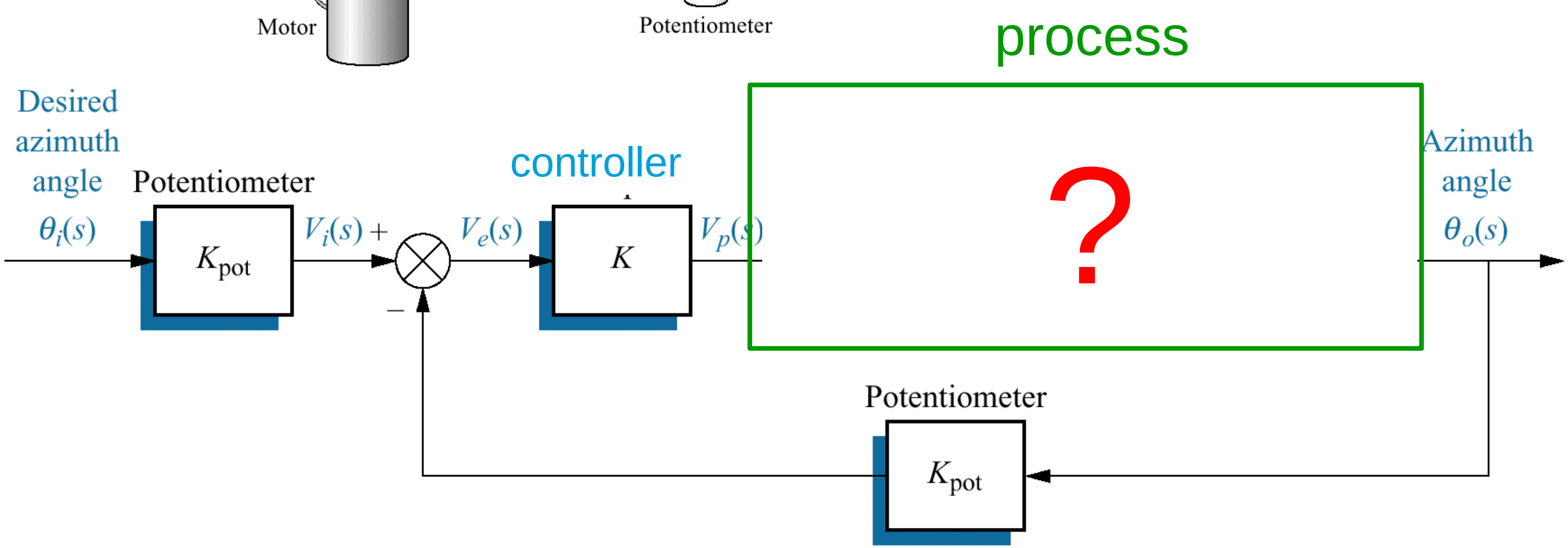
Where we do not know the transfer function.

But we can measure it’s input and output to approximate a transfer function.

Process analysis



By analysing the process we want to derive from the step response the mathematical model of the process and build the corresponding block diagram.



Process analysis

How does the output of the process change as the result of a stepwise change of the input to the process ?

This can be measured for the real system. From these measurements a model can be derived to approximate the dynamic behaviour.

Four types of models are used for most systems:

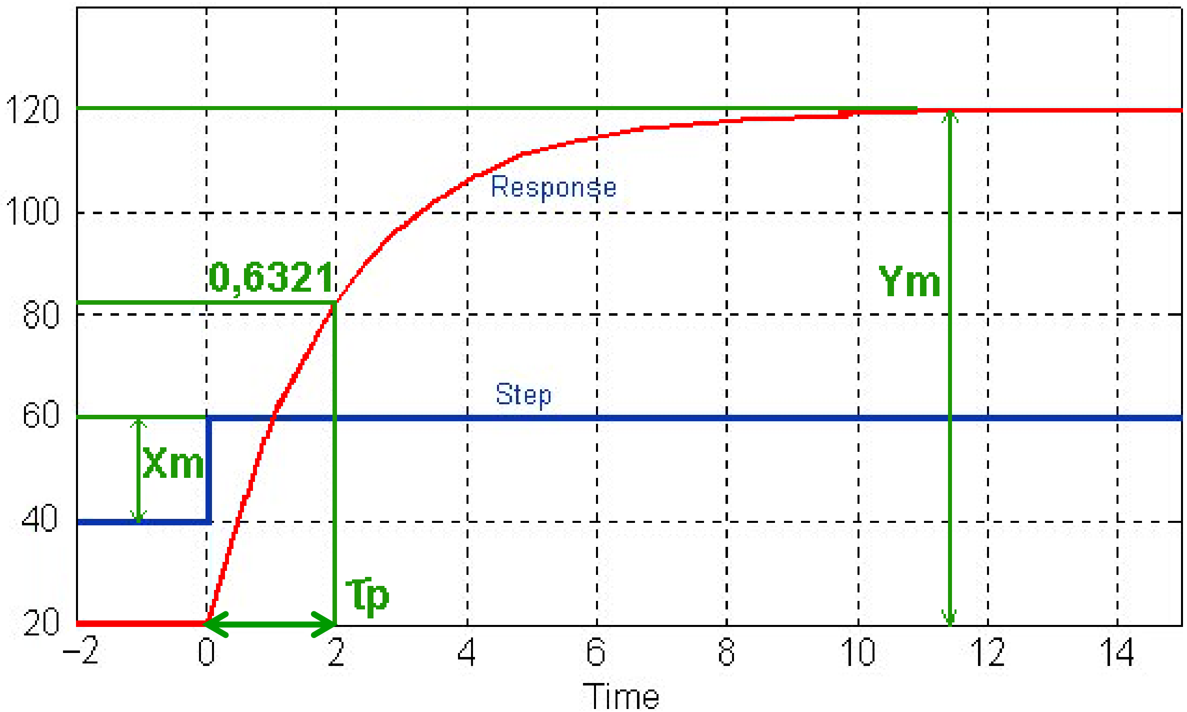
1. First order process
2. Delayed first order process
3. Second order process
4. Delayed second order process

In this presentation we will learn how to perform these approximations such that we can develop a controller for the most common processes.

1. First-order process

Step response of the system: $H_P(s) = \dfrac{K_P}{\tau_P s + 1}$

The output is given by: $y(t) = K_P(1 - e^{-\frac{t}{\tau_P}})$



steady state gain $K_P = \dfrac{Y_m}{X_m}$

time constant τ_p :
A characteristic time interval where

$$\boldsymbol{y(t_0) = \left(1 - e^{-1}\right) Y_m}$$

$$1 - e^{-1} \approx 0{,}6321$$

Note that $y(\tau_P) = K_P(1 - e^{-\frac{\tau_P}{\tau_P}}) \cdot X_m = (1 - e^{-1})Y_m = 0{,}6321 Y_m$

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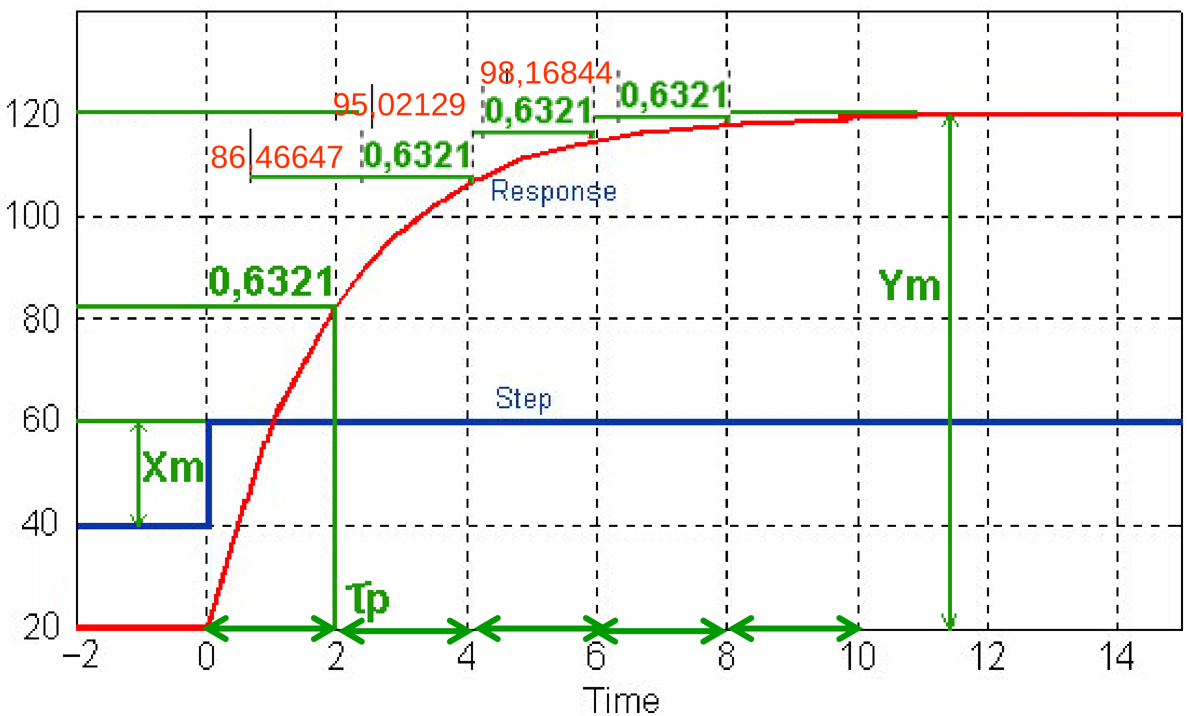

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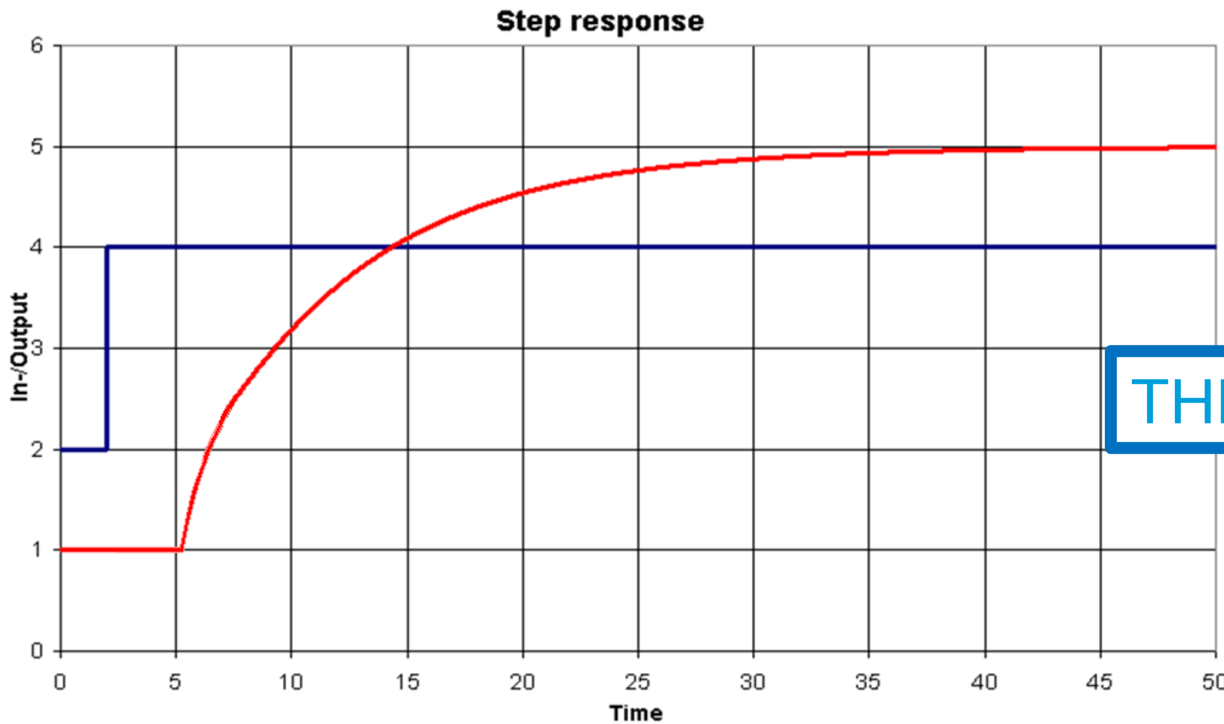
2. Delayed first order process

time shift

Step response of the system: $$H_P(s) = \frac{K_P e^{-\tau_v s}}{(1+\tau_p s)}$$

The output is thus the same as process 1 but it starts t_v seconds later

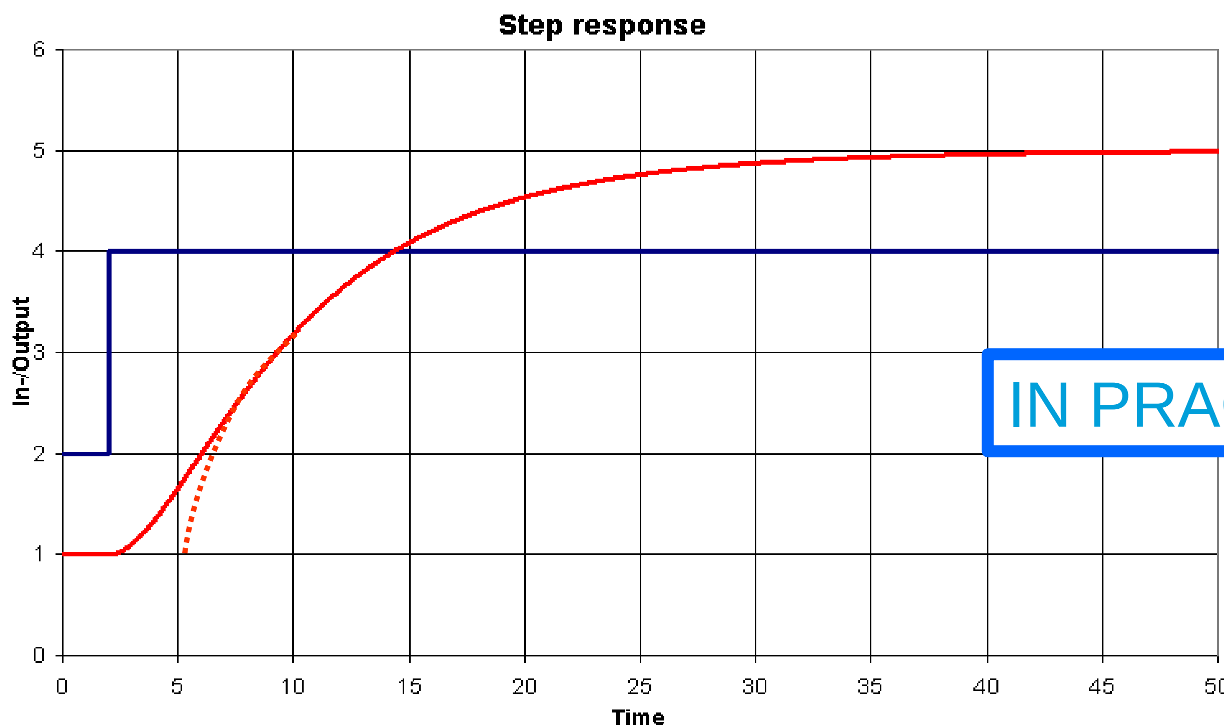


THEORY

2. Delayed first order process

Step response of the system: $H_P(s) = \dfrac{K_P e^{-\tau_v s}}{\tau_p s + 1}$ time shift

In practice it is often difficult to see at what time the output changes. The output will look more like the following:



IN PRACTICE

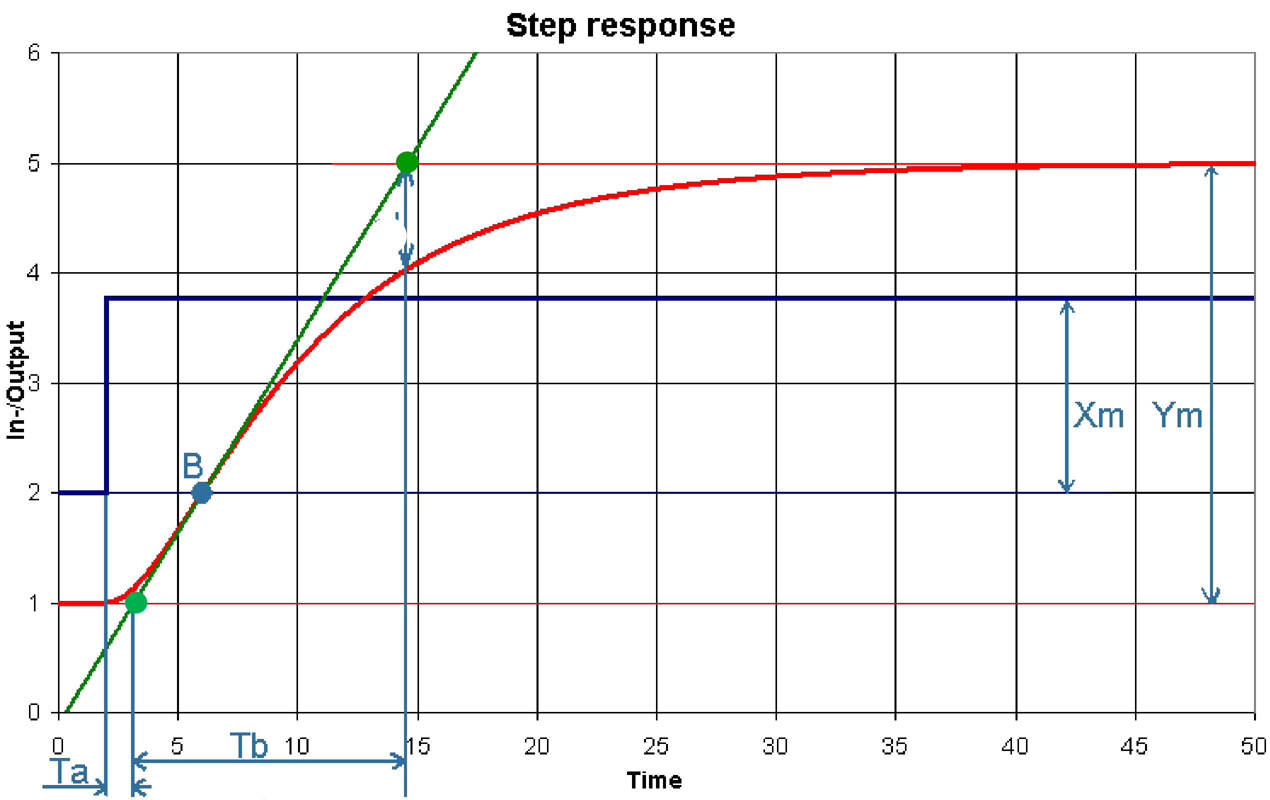
2. Delayed first order process

Method of approximation

1. Determine the point of inflection B

2. Draw tangent through the point of inflection

3. Determine X_m, Y_m, T_a, T_b



steady state gain $K_P = \dfrac{Y_m}{X_m}$

$$\tau_P = T_b$$

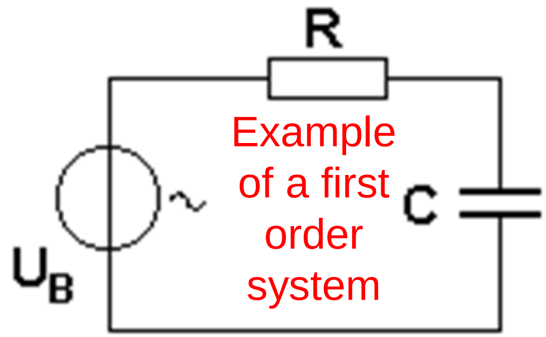
$$\tau_v = T_a$$

$$H_P(s) = \frac{K_P e^{-\tau_v s}}{\tau_p s + 1}$$

First order system

Calculate the output $y(t)$ for the following system with unit step input

$$H_P(s) = \frac{K_P}{\tau_P s + 1}$$



The entire system is defined by two parameters:

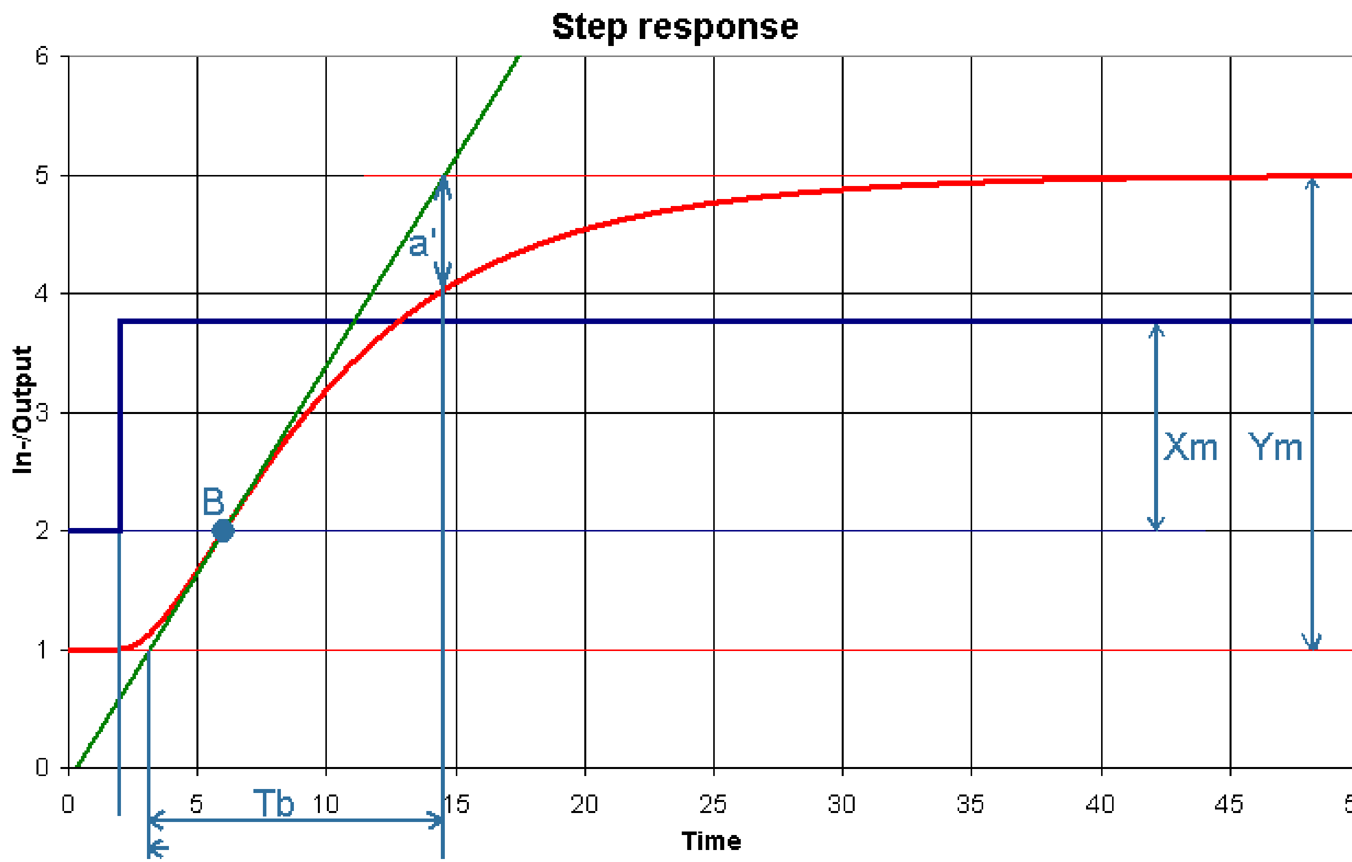
1. the steady state gain K_p
2. The time constant τ_p

$$y_{(t)} = K_p\left(1 - e^{-\frac{t}{\tau_P}}\right)$$

What is the relation between the pole location, the time constant and the speed of the system?

3. Second-order process: overdamped

Step response of the system: $H_P(s) = \dfrac{K_P}{(1+\tau_1 s)(1+\tau_2 s)}$



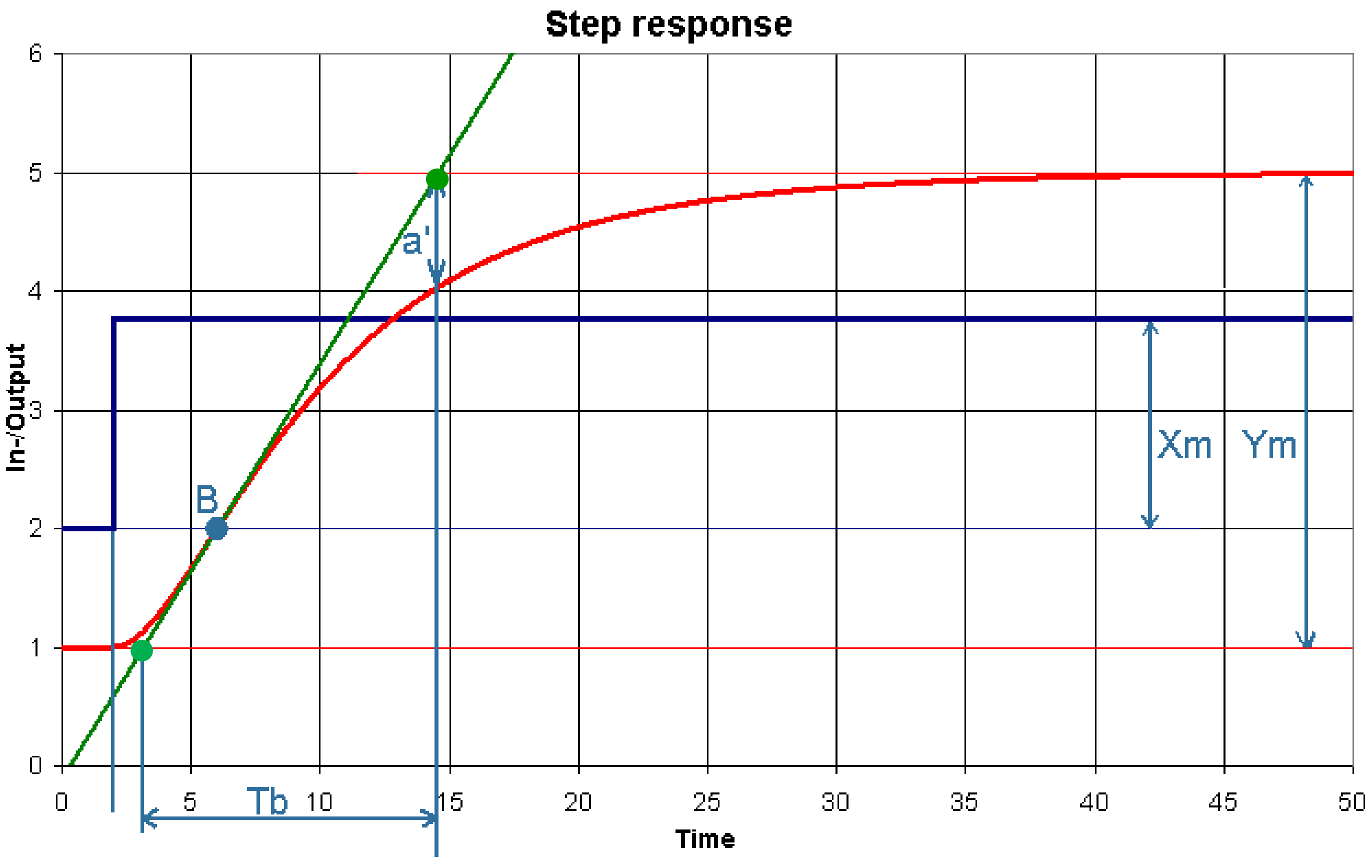
Note that there is little difference with the delayed first order process

3. Second-order process

Method of approximation

$$H_P(s) = \frac{K_P}{(1+\tau_1 s)(1+\tau_2 s)}$$

1. Determine the point of inflection B
2. Draw tangent through the point of inflection
3. Determine X_m, Y_m, T_b and a'



steady state gain $K_P = \frac{Y_m}{X_m}$

$$a = \frac{a'}{Y_m}$$

$$e \approx 2.71$$

$$\tau_1 = T_b \cdot \frac{3ae-1}{1+ae}$$

$$\tau_2 = T_b \cdot \frac{1-ae}{1+ae}$$

3. Second-order process

Method of approximation

Normally spoken it should be that: $\tau_1 > \tau_2$

If $\tau_1 < \tau_2$, as a result of (for instance):

- imprecise defined point of inflection
- inaccurate measurement
- time constants are too close to each other

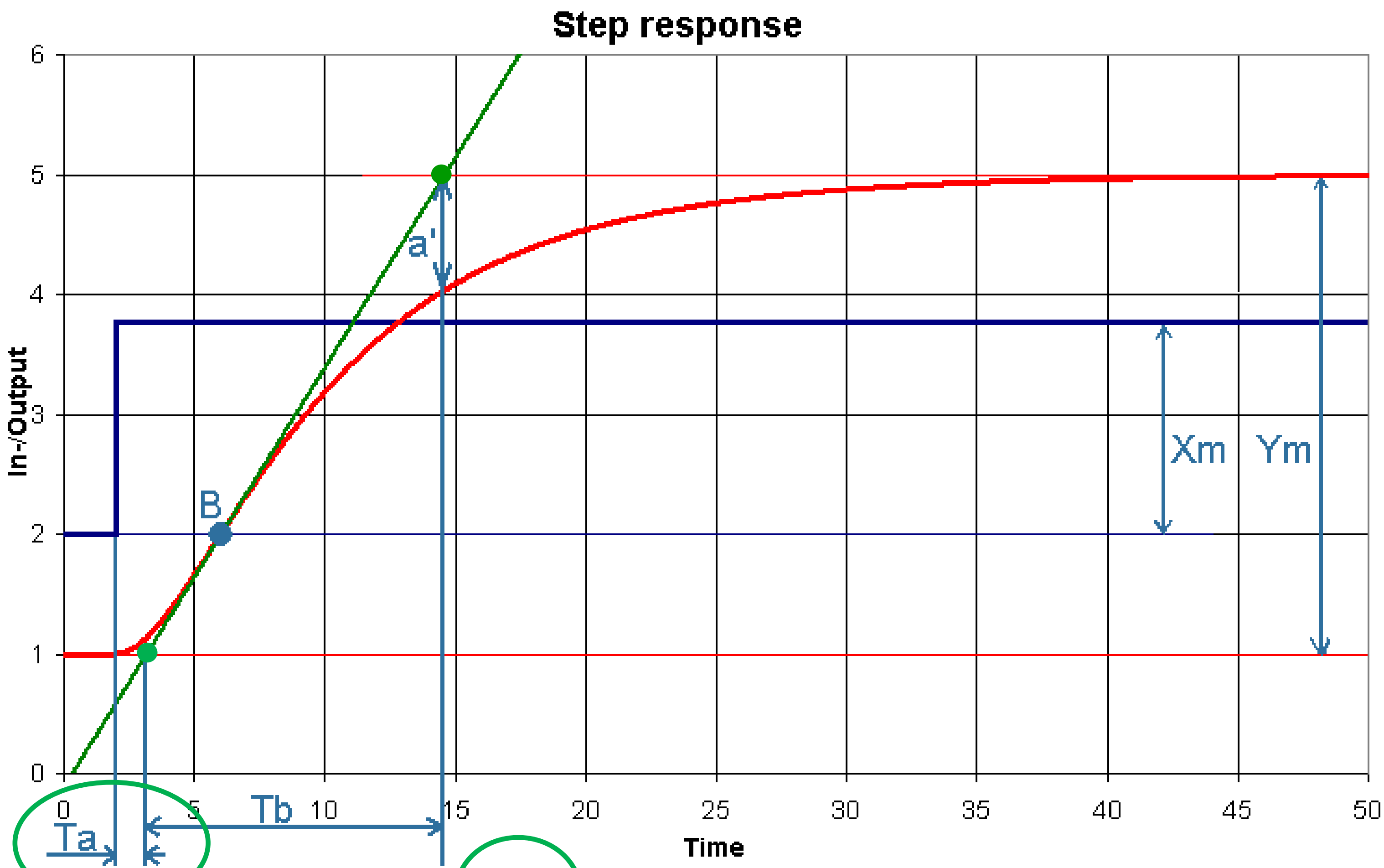
then the parameters of the process can still be determined in two different ways:

1. recalculate: $\tau_1 = \tau_2 = \dfrac{2\tau_1\tau_2}{\tau_1 + \tau_2}$

2. use an other process such as the delayed 1st order process or the delayed 2nd order process

4. Delayed second-order process

Method of approximation



$$H_P(s) = \frac{K_P \cdot e^{-s\tau_v}}{(1+\tau_1 s)(1+\tau_2 s)}$$

$$\tau_v = T_a - \frac{\tau_1 \tau_2}{\tau_1 + 3\tau_2}$$

HIGHER ORDER SYSTEMS

- Many processes are of higher order than 1st or 2nd
- Usually, you can determine the to most dominant time constants, τ_1 and τ_2.
- Higher order processes can often be considered as 2nd order processes, neglect the other time constants
- Compare the 2nd order model step response to the higher order actual response
- If the difference is small enough, accept the simplification
- If the difference can't be neglected, then modify the chosen time constants, or add an additional model order (3rd, 4th, etc.)

REAL LIFE ESTIMATION

Tools to choose about what you need: (Advanced)

Welch’s method:
for spectral density estimation using fast Fourier transform.

Pade approximations
Prony’s method

Nonlinear regression for linear combinations of exponential functions.

- Delay matrix Least Squares
- gradient descent
- quasi-Newton

Advanced tools provided by convex optimization and signal processing techniques

MODERN TOOLS IN THE AI ERA

Model Based Reinforcement Learning (MBRL)

Model Predictive Control (MPC)

Dynamic programming (DP)

SUMMARY

Time constant is a “magically” interesting number.

We can estimate system model from measured process data.

We know some advanced tools.

HOMEWORK

Stage ONE exercises:

- Problem 10

Test exam 3:

- Problem 4