A SOLUTIONS TO TEST EXAMS

A.A Solutions to test exam 1

Problem 1

Problem 1 (15 points)

Final TF: 7 pts;
correct 2 intermediate steps: 4 pts each;
small partial mistakes: 2 pts each.

Given the block diagram in Figure 1.

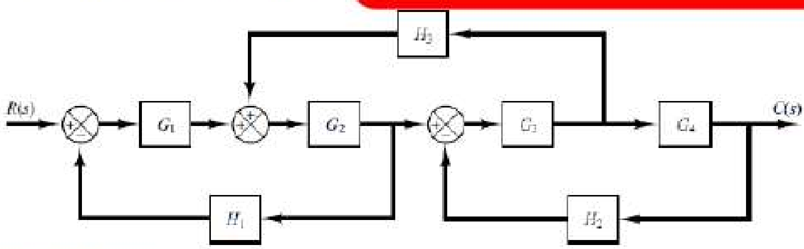
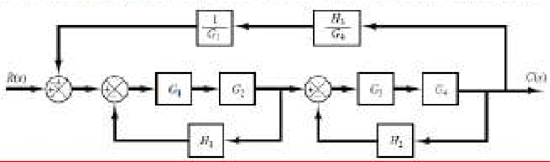


Figure 1 Block diagram

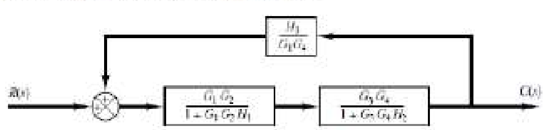
Find the transfer function H(s) = C(s)/R(s).

Show at least two intermediate steps used to find the solution.

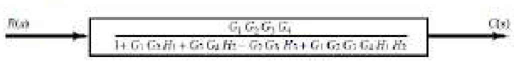
First move the branch point between G_3 and G_4 to the right-hand side of the loop containing G_3, G_4, and H_2. Then move the summing point between G_1 and G_2 to the left-hand side of the first summing point.



By simplifying each loop, the block diagram can be modified as



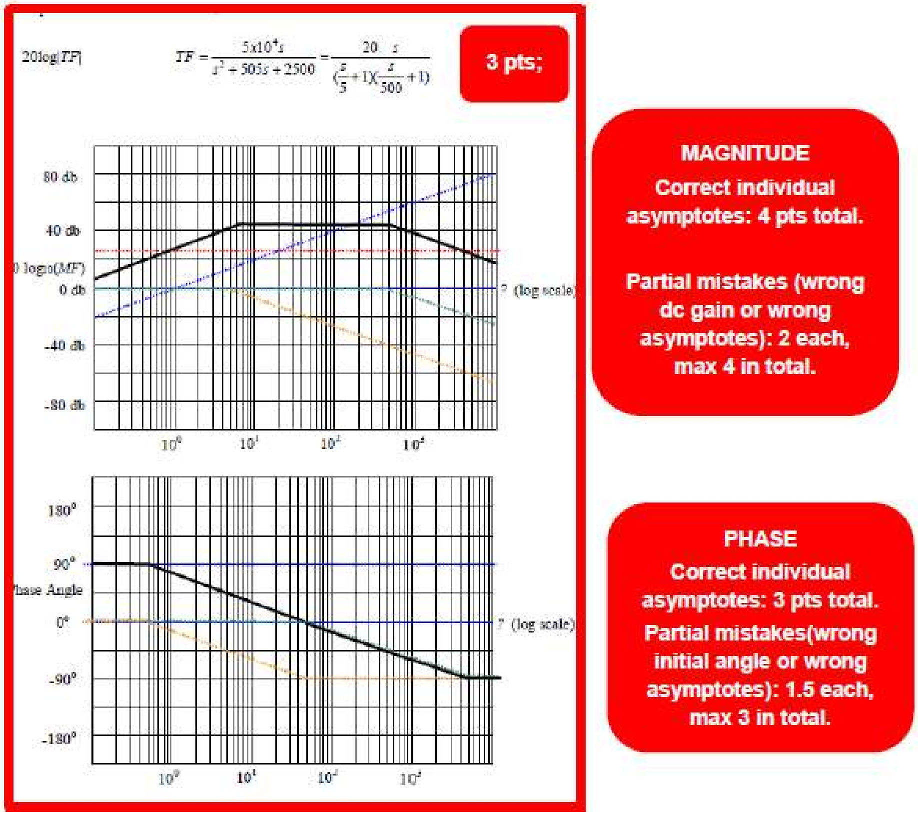
Further simplification results in



the closed-loop transfer function $C(s)/R(s)$ is obtained as

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1 + G_1G_2H_1 + G_3G_4H_2 - G_2G_3H_3 + G_1G_2G_3G_4H_1H_2}$$

Problem 2



20log|TF|
$TF = \frac{5x10^4 s}{s^2+505s+2500} = \frac{20 \quad s}{(\frac{s}{5}+1)(\frac{s}{500}+1)}$
3 pts;
80 db
40 db
0 db
-40 db
-80 db
(log scale)
10^0
10^1
10^2
10^3
180°
90°
Phase Angle
0°
-90°
-180°
(log scale)
MAGNITUDE
Correct individual asymptotes: 4 pts total.
Partial mistakes (wrong dc gain or wrong asymptotes): 2 each, max 4 in total.
PHASE
Correct individual asymptotes: 3 pts total.
Partial mistakes(wrong initial angle or wrong asymptotes): 1.5 each, max 3 in total.

Problem 3

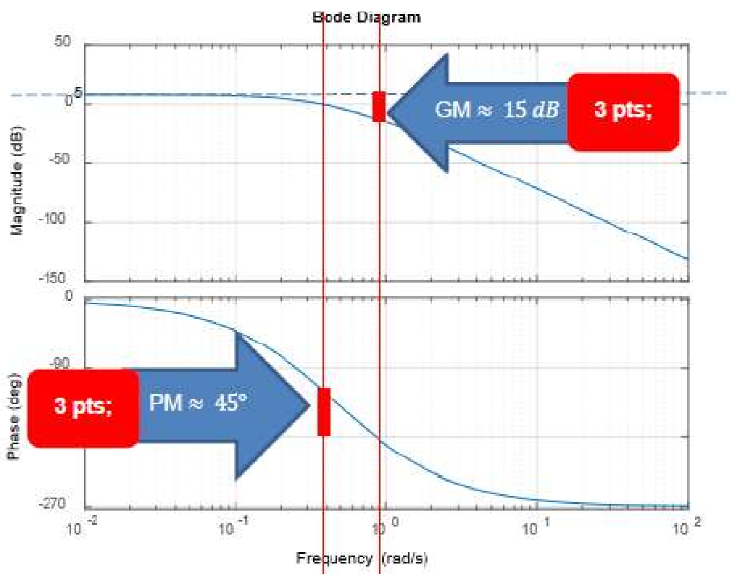


Figure 2 Bode plots of the transfer function G(s).

Find and clearly indicate the gain margin (GM) in dB and phase margin (PM) in degrees on the sketch sheet.

Determine whether the following ... are TRUE or FALSE:

3 pts;

a) The system is stable.
TRUE GM > 0, PM > 0

3 pts;

b) The DC gain of $G(s)$ is 0dB. FALSE, should be 5 dB

c) When the controller gain $K = GM + 10\,\mathrm{dB}$, the system will remain stable.
FALSE, the K would be too large such that the GM is -10dB, system unstable.

3 pts;

Fully motivate your judgement.

GM/PM sketch 2 pts each, value 1 pt each.
TRUE/FALSE problem: 3 pts each,
correct T/F judgement 1pt each,
sufficient explanation 2 pts each.

Problem 4

Estimating first-order process without delay from step response data

Parameters:

- DC gain K_P ,
- time constant τ_P .

The general form of a first-order process without delay with the parameters above:

$$H(s) = \frac{Y(s)}{X(s)} = \frac{K_P}{\tau_P s + 1}$$

3 pts;

Steps:

1. Find the gain of input X_{gain} and the steady-state gain of output Y_{gain}, then we may discover $K_p = \frac{Y_{gain}}{X_{gain}}$. **2 pts;**
2. Find the point when the output $y(t_A) \approx 0.6321 Y_{gain} + y(0)$. Then we may find $\tau_P = t_A$

Figure 3 The engineer's partially destroyed notebook

Answer the following questions:

a) Fill in the 'a' spot on the engineer's notebook
b) Fill in the 'b' spot on the engineer's notebook
c) Explain origin of this 'magical number' 0.6321 in step 2.

Note that $y(\tau_p) = K_p(1 - e^{-\frac{\tau_p}{\tau_p}}) \cdot X_m = (1 - e^{-1})Y_m = 0.6321 Y_m$ **5 pts;**

Problem 5

Impulse input so X(s) = 1 **2 pts;**

$$Y(s) = X(s)H(s) = \frac{s+10}{(s+10+j)(s+10-j)}$$ **1 pt;**

$$= \frac{A}{s+10+j} + \frac{B}{s+10-j}$$ **1 pts;**

$$\Rightarrow A+B = 1;\ 10A+10B = 1;\ A-B=0$$ **1 pt;**

$$A = B = \frac{1}{2}$$ **1 pt;**

$$y(t) = \frac{1}{2}e^{(-10-j)t} + \frac{1}{2}e^{(-10+j)t} = e^{-10t}\cos(t)$$

3 pts; **1 pt;**

Problem 6

Poles: -1, -8+5j, -8-5j; zeros: -5

P & Z identification 4 pts (1 pt each);

$$H(s) = K_0\frac{s+5}{(s+1)(s^2+16s+89)}$$

DC gain = $K_0\frac{5}{89} = 5 \Rightarrow K_0 = 89$ **2 pts correct dc gain;**

$$H(s) = \frac{89s+445}{s^3+17s^2+105s+89}$$

1 pt correct OLTF;

Closed-loop transfer function:

$$\frac{KH(s)}{1+KH(s)} = \frac{s+5}{s^3+17s^2+106s+94}$$

3 pts correct CLTF;

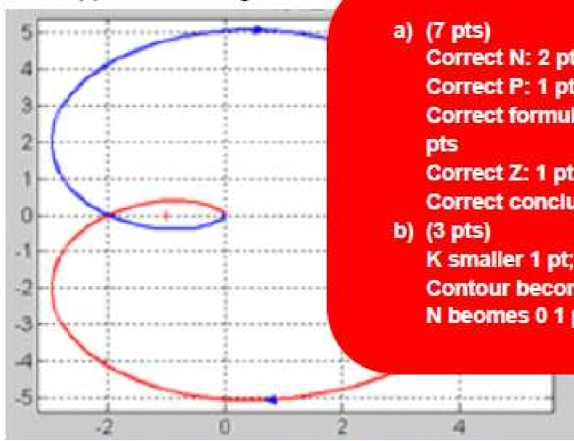
Problem 7

Problem 7 (10 points)

We have an open-loop process with transfer function $H(s)$ that have 3 poles:

- $s = -2$
- $s = -1 + 1.99j$
- $s = -1 - 1.99j$

The Nyquist plot of $H(s)$ is shown in Figure 6.



a) (7 pts)
Correct N: 2 pt
Correct P: 1 pt
Correct formula Z = N+P: 2 pts
Correct Z: 1 pt
Correct conclusion: 1 pt
b) (3 pts)
K smaller 1 pt;
Contour become smaller 1pt;
N beomes 0 1 pt.

Figure 6 Nyquist plot of the open-loop transfer function

Assuming a proportional controller K deployed to control the open-loop process with unity negative feedback loop in the closed loop system.

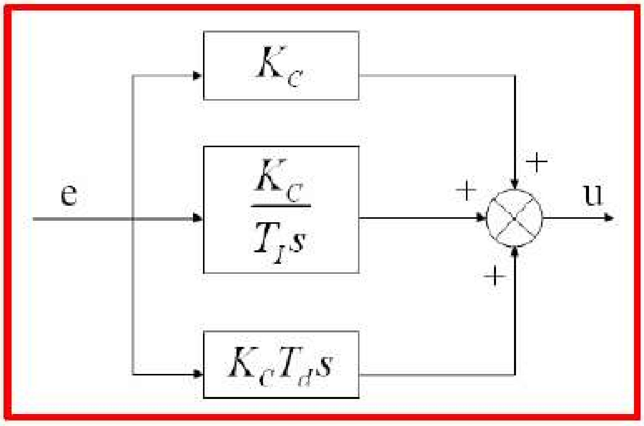
a) Determine the stability of the closed-loop system using Nyquist stability criteria.
b) What should we do to the proportional controller K to make the system stable? Analyze using the property of the Nyquist plot, do not use explicit calculation.

Z = N + P; from the graph we see 2 clock wise encirclement about -1 point. Thus N = 2. There are no open-loop poles in right half plane, thus P = 0.

Z = 2, thus unstable!

Make K smaller than 1, such that the coutour shrinks to the extent that -1 point is outside of the contour. Then we have N = 0. With P = 0, Z=0 then the system is stable.

Problem 8



5 pts;

$$H_C(s) = K_c\left(1 + \tau_d s + \frac{1}{\tau_i s}\right)$$

5 pts;

Or the student can multiply the K_c in such that we have 3 parameters Kp, Ki, and Kd.

Problem 9

$$U_{in}(t) = \mathrm{RC}\frac{\mathrm{d}U_{out}}{\mathrm{dt}} + U_{out}(t)$$

1 pt;

$$U_{in}(s) = \mathrm{RC}sU_{out} + U_{out}(s)$$

2 pts;

$$U_{in} = \mathrm{RC}sU_{out} + U_{out}$$

$$H(s) = \frac{1}{1 + RCs}$$

2 pts;

$$\mathrm{BW} = \frac{1}{RC}$$

2 pts;

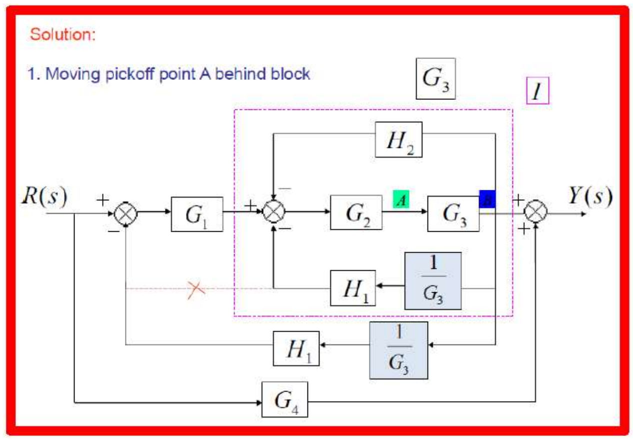
$$C = \frac{1}{BW \cdot R} = 10^{-6}\,F = 1\,\mu F$$

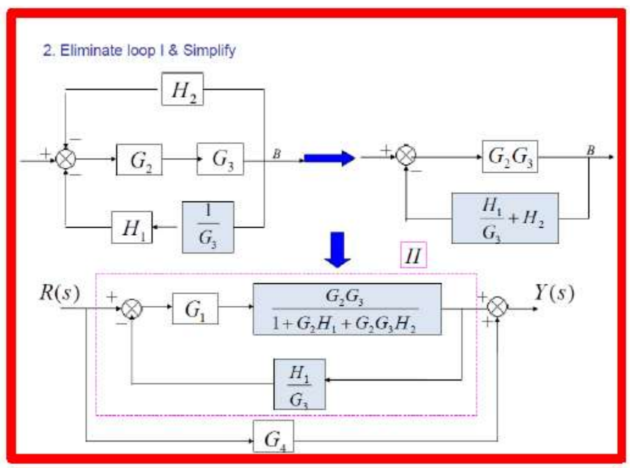
2 pts;

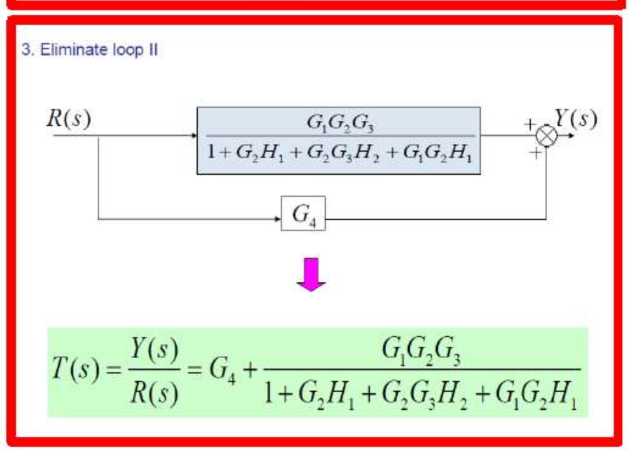
A.b Solutions to test exam 2

Problem 1

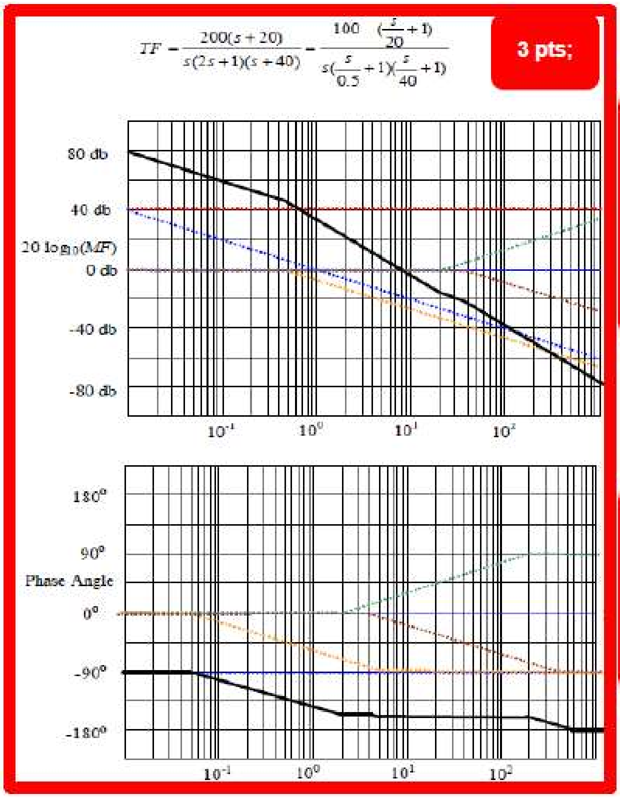
Final TF: 7 pts;
correct 2 intermediate steps: 4 pts each;
small partial mistakes: 2 pts each.







Problem 2



MAGNITUDE
Correct individual asymptotes: 4 pts total.

Partial mistakes (wrong dc gain or wrong asymptotes): 2 each, max 4 in total.

PHASE
Correct individual asymptotes: 3 pts total.
Partial mistakes(wrong initial angle or wrong asymptotes): 1.5 each, max 3 in total.

Problem 3

Problem 3 (15 points)

Given the following ordinary differential equation:

$$\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 10\frac{dy}{dt} = \frac{d^2x(t)}{dt^2} - 3x(t)$$

Assume ZERO initial condition.

a) Determine the transfer function $H(s) = \frac{Y(s)}{X(s)}$.

b) Determine the poles and zeros of $H(s)$.

c) Visualize the poles and zeros in the complex plane.

transfer function $\frac{s^2-3}{s^3+6s^2+10s}$

4 pts;

Transfer function element zeros

$s = -\sqrt{3}$ (simple zero)

$s = \sqrt{3}$ (simple zero)

Transfer function element poles

$s = -3 - i$ (simple pole)

$s = -3 + i$ (simple pole)

$s = 0$ (simple pole)

C) 3.5 pts;
0.5 pt each for correct poles and zeros in the s-plane;
1 pt for fully correct axis labels, not full correct, no points

b) 2.5 pts;
0.5 pt each for correct poles and zeros

Given the following bode plots.

d) Out of the plots 1, 2, and 3, which one represents the transfer function derived from the given ordinary differential equation? Why?

3

The pole at the origin gives the system magnitude plot an initial -20dB drop.

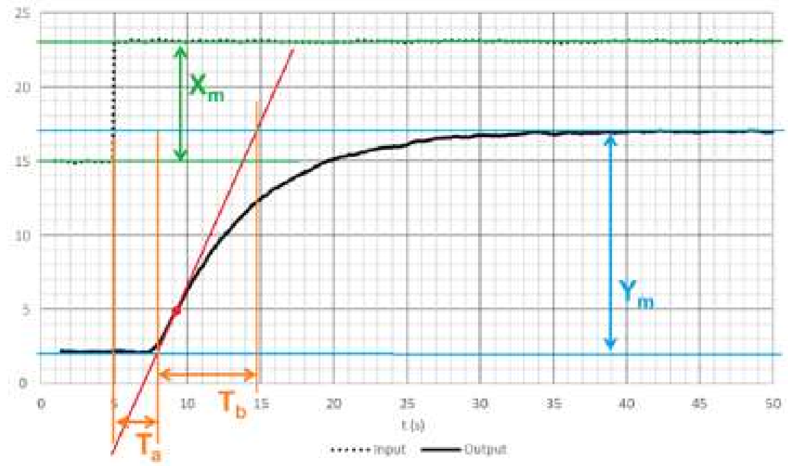
The system tends to decrease to 0 as frequency becomes very large.

3 pts;
Make correct selection 1 pt.
Reasoning 2 pts.
Student only need to mention one valid reason to get full mark.

e) How large is the gain margin of this system? ∞

2 pts;
It's fine if student only answer infinity without explanation

Problem 4



$K_P = Y_m/X_m = (17\text{-}2)/(23\text{-}15) = 1.875 \rightarrow X_m$ and Y_m noted in the graph and found correctly is **2 points**, K_P calculated correctly is **1 points**.

$T_a = \tau_v = 8\text{-}5$ s $= 3$ s (±10% error margin for reading from the graph) is **2 points**
$T_b = \tau_P \approx 15\text{-}8 \approx 7$ s (range from 6 to 8 s is correct) is **2 points**

3 points for whole transfer function.

$$H_p(s) = \frac{K_P\ e^{-\tau_v s}}{\tau_P\, s+1} = \frac{1.875\ e^{-3s}}{7s+1}$$

Problem 5

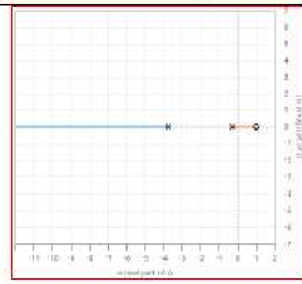
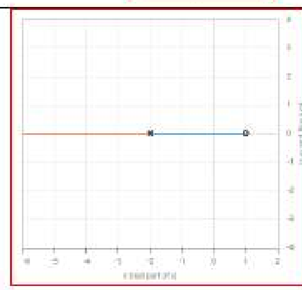
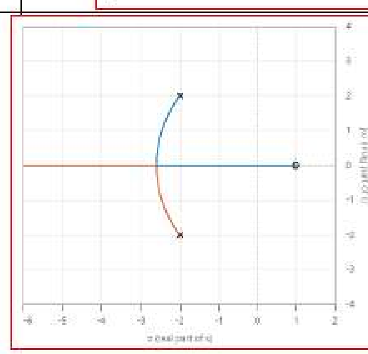
3 * 8 = 24.

3 systems, for each system: 8 pts.

Poles	Zeros	Root Locus	Time domain	Matching
1 pt	0.5 pt	2.5 pts	2.5 pts	Selection: 1 pt Reasoning 0.5 pt

For each item, only grant full points when fully correct, otherwise zero points granted.

Reasoning: as long as reasonable, we may grant point.

System	Zero	Pole	Time domain	Graphs
H	1	-2+sqrt(3); -2-sqrt(3)	$\dfrac{3\,e^{(-2-\sqrt{3})t} + \sqrt{3}\,e^{(-2-\sqrt{3})t} - 3\,e^{(\sqrt{3}-2)t} + \sqrt{3}\,e^{(\sqrt{3}-2)t}}{2\sqrt{3}}$	3
Root Locus H(s)				
G	1	-2; -2	$-e^{-2t}(3t-1)$	1
Root Locus G(s)				
M	1	-2 + 2j; -2 - 2j	$\frac{1}{2}e^{-2t}(2\cos(2t) - 3\sin(2t))$	2
Root Locus M(s)				

Problem 6

(a)	(b)	(c)
Z= N + P. P = 1 (1 pts) N = 1 (1 pts) Z = 1 + 1 = 2 (1 pts) Unstable! (1 pts)	Z= N + P. P = 0 (1 pts) N = 1 (1 pts) Z = 1 + 0 = 1 (1 pts) Unstable! (1 pts)	Z= N + P. P = 0 (1 pts) N = 0 (1 pts) Z = 0 + 0 = 0 (1 pts) Stable! (1 pts)

Problem 7

Problem 7 (8 points)

Consider a PID controller with the parallel structure. Answer the following questions and motivate your answer in less than 2 sentences for each question: (You can use drawings when you think it's easier to explain.)

a) State one reason why we can never use the D controller alone.
Error no change, controller no change.
Sensitive to noise, might amplify the noise.
Sensitive to disturbance, causing the system to overreact and become unstable.
Small sampling time causing large fluctuation.
Etc.....

b) Which controller eliminates the steady state error?
I – controller. Because the integration action eventually eliminates the accumulated error.

c) Will higher gain of the I controller give you higher overshoot when P and D remains the same?
Yes (or very probable). The wind-up effect. (Or anything similar/related to describing the wind-up effect also counts as correct answer. Or just state over compensation of accumulated error)

d) I'm using one of the P,I, and D controller to control a process. The step response of the controlled process is plotted in Figure 7. Which controller am I using?
D controller. The response time is very high, and it has a high overshoot and oscillation problems as well as steady-state errors. Damping effect is rather obvious.

a: 2 pts.

b to d: Answer 1 pt, explanation 1 pt.)

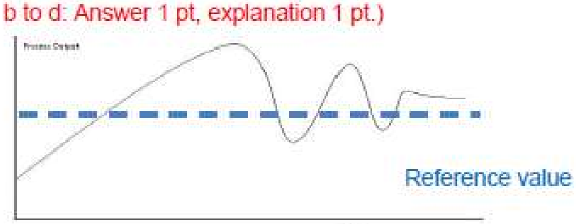


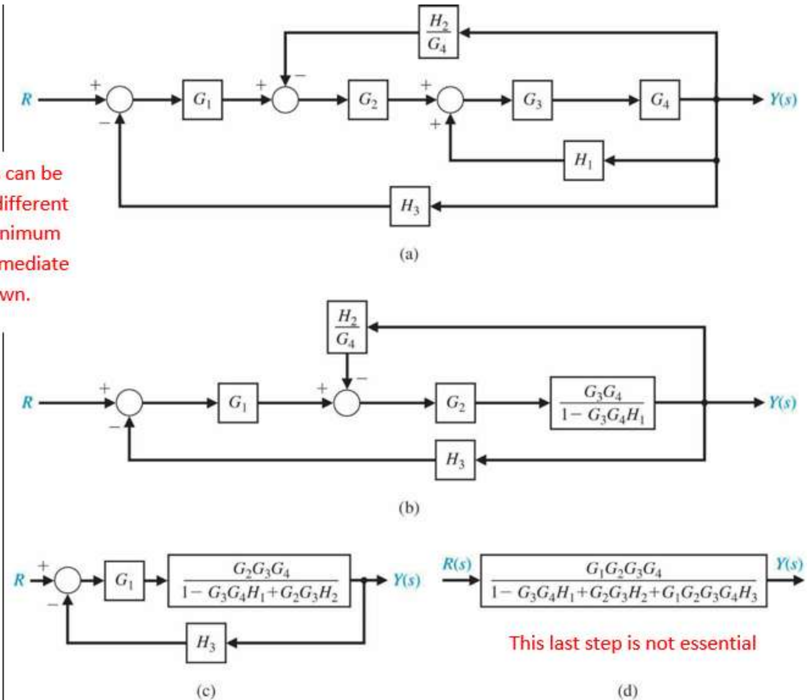
Figure 7 Step response of the controlled process

A.C Solutions to test exam 3

Problem 1

Answer 1
(15 points)

The steps can be taken in different order. Minimum of 2 intermediate steps shown.



Grading instructions:
15 points: Correct method without errors/different method with the same answer
12 points: Correct method with only minor mistake in answer (sign error, calculation error).
12 points: Correct method and answer but minor mistakes in intermediate steps.
10 points: Correct method and answer but TF still contains 1/s terms.
<10 points: Everything else, at the discretion of the examiner.

Problem 2

Exam A Q2

Rewrite the function in its standard form:

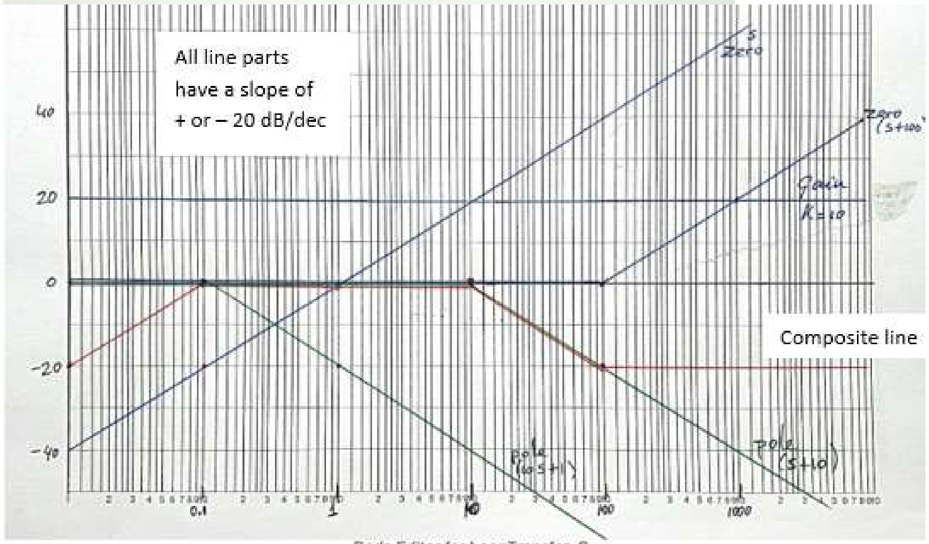
$$H(s) = \frac{100}{10} \cdot \frac{s\left(\frac{1}{100} \cdot s + 1\right)}{(10s+1)\left(\frac{1}{10}s+1\right)}$$

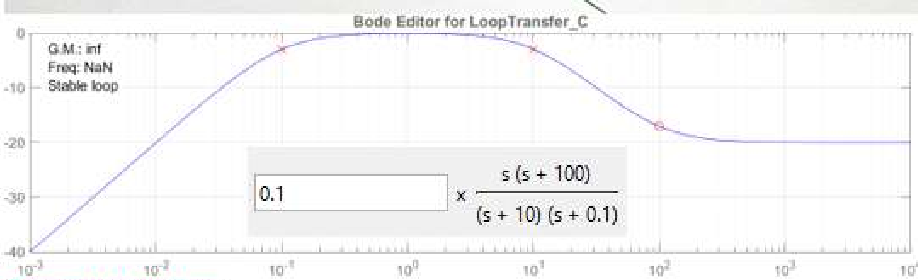
Then sketch the magnitude lines for the separate parts in the BD:

gain $K = 10 \Rightarrow M = 20$

zero's : 0 and -100 $\Rightarrow$ corner freq. n/a and 100

poles : -0.1 and -10 $\Rightarrow$ corner freq. 0.1 and 10





Grading instructions:

10 points: Correct method without errors. Different parts that add up to the complete function composite shown separately (or in separate sketches), slopes indicated. It is not needed to work out all details using the limit method.

5 points: correct shape of parts and composite, but no slopes indicated or unclear corner frequencies.

< 5 points: Everything else, at the discretion of the examiner.

Problem 3

The input is a step function which has as Laplace transform X(s) = 1/2. The output in the s-domain can then be written as

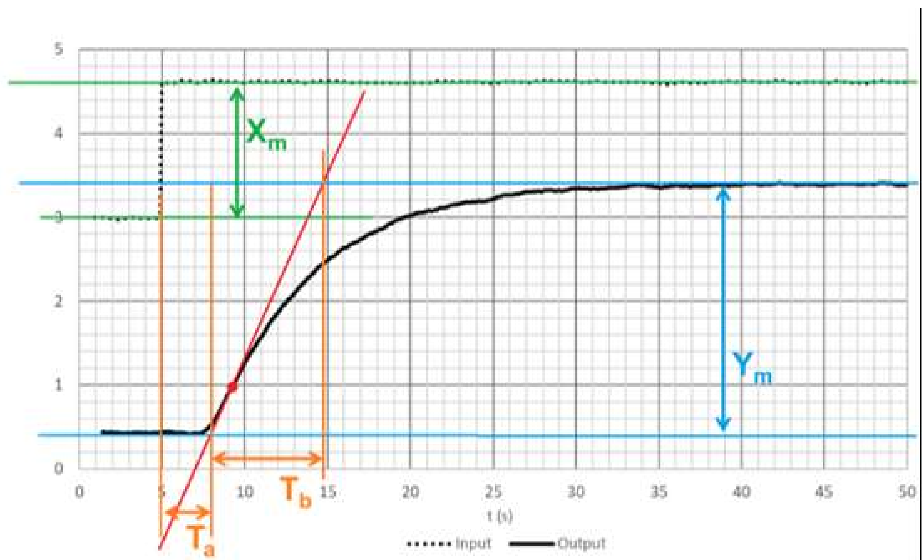
$$Y(s) = H(s)X(s) = H(s) = \frac{6s^2 + 2s + 8}{(s^2 + 2s + 4)s} = \frac{A}{s} + \frac{B(s+1) + C}{(s+1)^2 + 3}$$

The values of A,B and C can be calculated by writing both fractions again as one fraction: A=2, B=4 and C=-6. Furthermore it should be noticed that the last part corresponds to a cosine and a sine. Using tables 3 and 4 with the inverse Laplace transforms, one finds

$$y(t) = [2 + e^{-t}(4\cos(\sqrt{3}t) - \frac{6}{\sqrt{3}}\sin(\sqrt{3}t)]1(t)$$

Grading instructions:
5 points for proper expansion in fractions,
5 points for the Laplace transform of these fractions,
calculation error is 2 points deduction

Problem 4



$K_P = Y_m/X_m = (3.4-0.4)/(4.6-3) = 1.875$ → X_m and Y_m noted in the graph and found correctly is **3 points**, K_p calculated correctly is **2 points**.

$T_a = \tau_v$ = 8-5 s = 3 s (±10% error margin for reading from the graph) is **4 points**
$T_b = \tau_p \approx$ 15-8 ≈ 7 s (range from 6 to 8 s is correct) is **4 points**

2 points for whole transfer function.

$$H_P(s) = \frac{K_P\, e^{-\tau_v s}}{\tau_p\, s + 1} = \frac{1.875\, e^{-3s}}{7s + 1}$$

Problem 5

From PZ-map:

2 zero's at +2+j and +2-j

3 poles at -3, -2+j and -2-j

From here work out the TF:

$$sys = K \cdot \frac{s^2 - 4s + 5}{s^3 + 5s^2 + 11s + 15} = \frac{Y(s)}{X(s)}$$

This gives:

$$Y(s) \cdot (s^3 + 5s^2 + 11s + 15) = X(s) \cdot K \cdot (s^2 - 4s + 5)$$

The DC-gain equals 3: for s=0 then sys=3=5K/15 → K=9

This gives the DE:

$$\frac{d^3y(t)}{dt^3} + 5\frac{d^2y(t)}{dt^2} + 11\frac{dy(t)}{dt} + 15y(t) = 9\left(\frac{d^2x(t)}{dt^2} - 4\frac{dx(t)}{dt} + 5x(t)\right)$$

Matlab script:

```
>> numG = [1 -4 5];
>> denG = [1 5 11 15];
>> sys=tf(numG, denG)
>> pzmap(sys)
sys =
     s^2 - 4 s + 5
  -----------------------
  s^3 + 5 s^2 + 11 s + 15
```

Grading instructions:

10 points: fully correct solution incl. sufficient explanation (P&Z, TF, DC-gain)

8 points: as 10 points but minor calculation errors or mistakes in steps.

7 points: as 10 points but DC-gain missing

5 points or less: everything else, at the discretion of the examiner.

Problem 6

Answer 6 (10 points)

a) 6 pts

Exam A Q6

a) At the frequency 0.25 we find from the Bode plot the values

$M = +10$

$\varphi = -160^\circ = -8/9 \cdot \pi$

A magnitude of 10 gives a gain of

$K = 10^{10/20} = 3.2$

therefor the output will be

$y(t) = 3.2 \cdot \cos(0.25t - 8/9 \cdot \pi)$

b) 4 pts

b) The system is stable, since at the frequency where the magnitude equals zero (approx. 0.8 rad/s) the phase is equal to -170 degree.

Grading instructions:

6+4=10 points: fully correct solution incl. correct explanations. Note that inaccuracies reading values from the graph are not counted as error, as long as the correct location is indicated in the graph (or described in sufficient detail).

Wrong frequency chosen and therefor wrong answer: deduct 2 points (both a and b)

No or wrong explanation with correct answer: deduct 2 points (both a and b)

Problem 7

Nyquist stability criterium

Observe the Nyquist plot

$$Z_{RHP} = N_{CWE} + P_{OL\_RHP}$$

the closed-loop system is unstable if Z > 0

Z_{RHP} = Number of zeros in the Right Half Plane

N_{CWE} = Number of Clock Wise Encirclements of the point $-1 + j0$

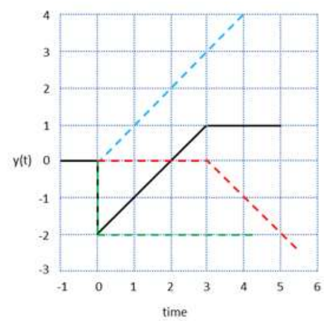
P_{OL_RHP} = Number of poles of the Open Loop system in the Right Half Plane

If encirclements are in the counterclockwise direction, N_{CWE} is negative

We have 2 (complex) poles in the RHP,
and 1 clockwise encirclement.
Therefor Z=1+2=3
and the system is **unstable**

Grading instructions:
10 points: fully correct solution incl. correct explanation.
5 points: correct use of criterium Z=N_{cwe} + P_{OL_RHP} but incorrectly seen only one encirclement counterclockwise.
< 5 points: everything else, at the discretion of the examiner.

Problem 8



$$y(t)\begin{cases} 0, & t < 0 \\ -2, & t \geq 0 \end{cases} \xrightarrow{\mathcal{L}} -\frac{2}{s}e^{-s}$$

$$y(t)\begin{cases} 0, & t < 0 \\ t, & t \geq 0 \end{cases} \xrightarrow{\mathcal{L}} +\frac{1}{s^2}e^{-s}$$

$$y(t)\begin{cases} 0, & (t-3) < 0 \\ -(t-3), & (t-3) \geq 0 \end{cases} \xrightarrow{\mathcal{L}} -\frac{1}{s^2}e^{-3s}$$

$$y(t) \xrightarrow{\mathcal{L}} \mathrm{Y(S)} = \xrightarrow{\mathcal{L}} \frac{1}{s}\left(\frac{1}{s} - 2e^{-s} - \frac{1}{s}e^{-3s}\right)$$

Grading instructions:
10 points: fully correct solution incl. correct explanation of separate parts (either in words or in drawing).
6 points: incorrect use of time shift in otherwise correct solution.
5 points: incorrect transform to Laplace domain.

Problem 9

a) To obtain a smoother response generally the gain should be lowered. This also will give a somewhat slower response, and an even larger steady state error.

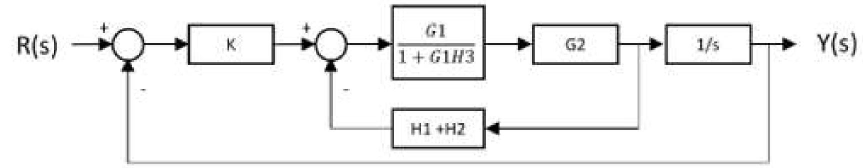
b) To remove the steady state error an integral action should be added to the controller. A P-only controller (with only gain K) will never reach a zero steady state error due to the nature of the feedback loop (sketch – error zero means zero input in controller block). The integral action will remedy this.

Grading instructions:
a: 6 points: fully correct answer incl. correct explanation of (unwanted) side effects.
b: 4 points: correct answer but without explanation.
Only half points for correct answer without any explanation.

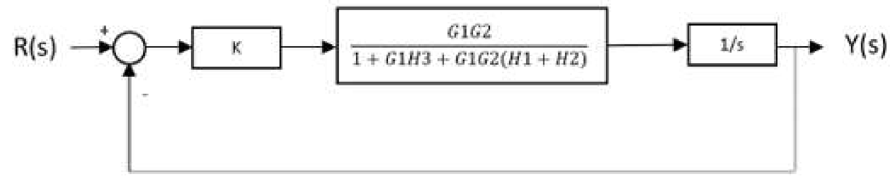
A.D Solutions to test exam 4

Problem 1

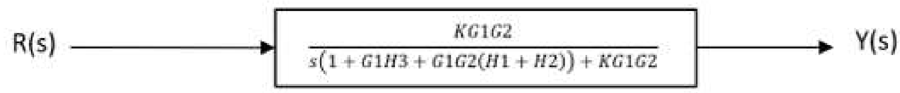
Simplify the block diagram step by step (minimum two steps, order may be different):
Step 1a: Combine the negative feedback loop that involves G1 and H3
Step 1b: Combine H1 and H2 they are in parallel and can be added



Step 2: Combine the remaining inner negative feedback loop



Step 3: Combine the remaining negative feedback loop



Grading instructions:
15 points: Correct method without errors.
12 points: Correct method with only minor mistake in answer (sign error, calculation error).
12 points: Correct method and answer but minor mistakes in intermediate steps.
<10 points: Everything else, at the discretion of the examiner.

Problem 2

Answer 2

a) 3 pts

a) Laplace transform on the differential equation gives:

$$3s^3Y(s) + 6s^2Y(s) + 15sY(s) = sX(s) + 4X(s)$$
$$Y(s)\{3s^3 + 6s^2 + 15s\} = X(s)\{s + 4\}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s+4}{3s^3 + 6s^2 + 15s} = \frac{1}{3}\frac{(s+4)}{s(s+1+2j)(s+1-2j)}$$

This last step is not essential

b) 4 pts

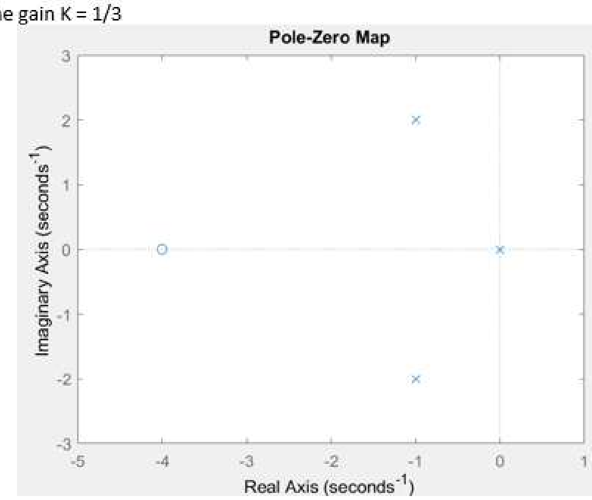
b) Find the 3 poles: $3s^3 + 6s^2 + 15s = 0$

p1 = 0; p2 = -1-2j; p2 = -1+2j;

Find the 2 zeros: $s + 4 = 0$

z1 = -4;

The gain K = 1/3



c) 3 pts

c) Find the DC gain H(s)=H(0)= 4/0 = infinite.
The DC gain (the gain at ω = 0 rad/s) is infinite because there is a pole in the origin.

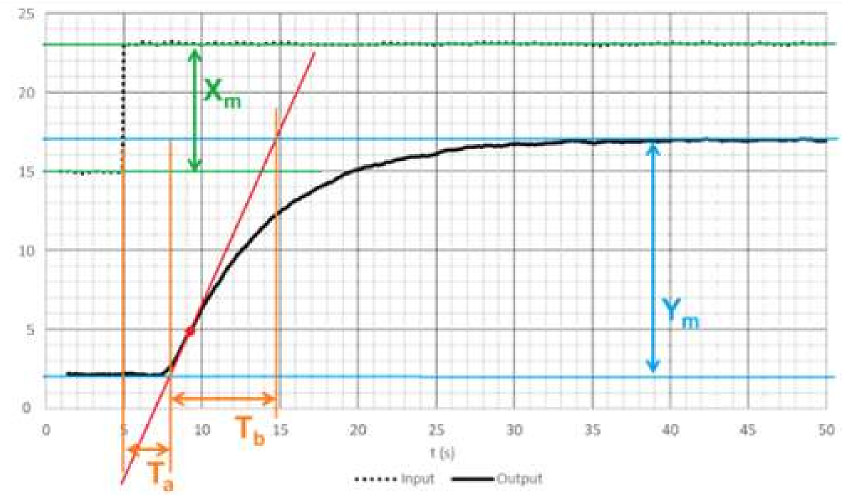
Grading instructions:

10 points: Correct methods and correct answers

Deduct 2 points for minor calculation error (sign) in a) or b) (do not deduct twice for same error).

Deduct 2 points if not clearly indicated that the zero is a double zero.

Problem 3



$K_p = Y_m / X_m = (17-2)/(23-15) = 1.875$ → X_m and Y_m noted in the graph and found correctly is **3 points**, K_p calculated correctly is **2 points**.

$T_a = \tau_v = 8\text{-}5$ s = 3 s (±10% error margin for reading from the graph) is **4 points**
$T_b = \tau_p \approx 15\text{-}8 \approx 7$ s (range from 6 to 8 s is correct) is **4 points**

2 points for whole transfer function.

$$H_p(s) = \frac{K_p\, e^{-\tau_v s}}{\tau_p\, s + 1} = \frac{1.875\, e^{-3s}}{7s + 1}$$

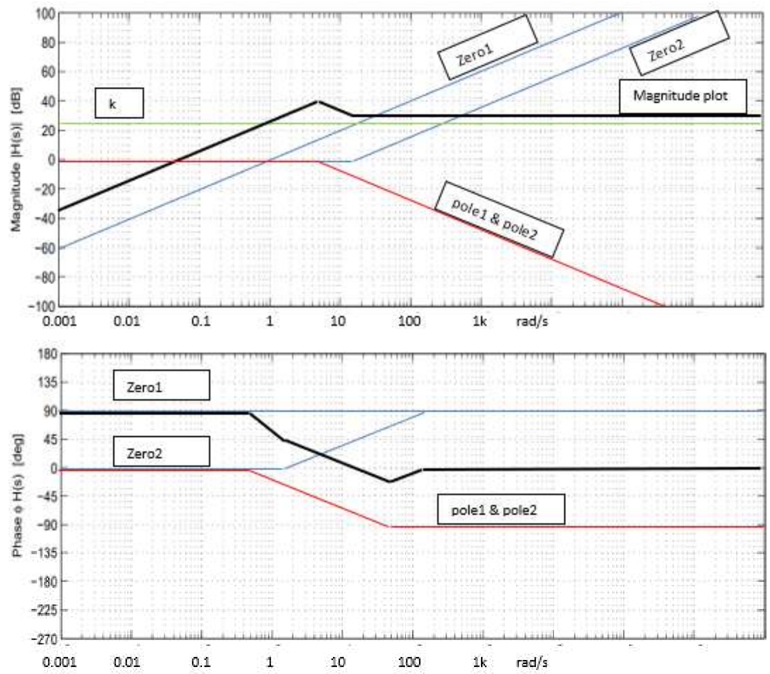
Problem 4

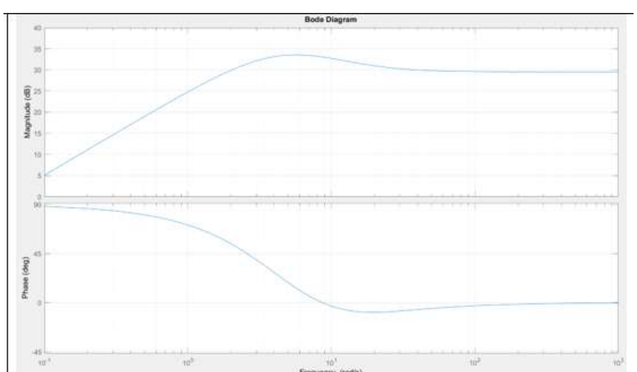
Substitute $j\omega$ for s and rewrite in the proper form:

$$H(s) = \frac{30 \cdot 15 \cdot j\omega\left(\frac{1}{15}s + 1\right)}{5 \cdot 5\left(\frac{1}{5}j\omega + 1\right)^2} = 18 \cdot \frac{j\omega\left(\frac{1}{15}j\omega + 1\right)}{\left(\frac{1}{5}j\omega + 1\right)^2}$$

Split into gain, poles and zeros:

1. gain H(0) = 0 → convert into dB is 20log(0)= - ∞
2. The frequency independent gain k = 18x convert into dB is 20log(18)= 25.1dB
3. Zero1 @ $\omega = 0\ rad/s$, Zero2 @ $\omega = 15\ rad/s$
4. Pole1 @ $\omega = 5\ rad/s$, Pole2 @ $\omega = 5\ rad/s$,





Grading instructions:

15 points: Correct method without errors. Different parts that add up to the complete function composite shown separately (or in separate sketches), slopes indicated. It is not needed to work out all details using the limit method.

Deduct 5 points if phase plot is incorrect.

Deduct 5 points if correct shape of parts and composite, but no slopes indicated or unclear corner frequencies. If the slope are clear from the logarithmic graph that is also fine.

Deduct remaining point at the discretion of the examiner.

Problem 5

Answer 5

a) 6 pts

a) Below follows one of the possible methods to find the Lapace transform of H(s).

Use: $s^2 + 2s + 7 = (s+1)^2 + 6$

$$H(s) = \frac{2(s+1)+2}{(s+1)^2+6}$$

$$h(t) = \mathcal{L}^{-}\left\{\frac{2(s+1)}{(s+1)^2+6} - \frac{2}{(s+1)^2+6}\right\}$$

Use the entries below from the laplace table:

Damping	$e^{-at}\cdot f(t)$	$F(s+a)$
	$sin(\omega t)\cdot 1(t)$	$\frac{\omega}{s^2+\omega^2}$
	$cos(\omega t)\cdot 1(t)$	$\frac{s}{s^2+\omega^2}$

$$h(t) = 2e^{-t}\cos(\sqrt{6}t) + \frac{2}{\sqrt{6}}e^{-t}\sin(\sqrt{6}t)$$

$$h(t) = e^{-t}\left[2\cos(\sqrt{6}t) + \frac{2}{\sqrt{6}}\sin(\sqrt{6}t)\right]$$

b) 4 pts

b) The overshoot of G(s) is calculated from β.

There are two probable ways the student will try to calculate β.

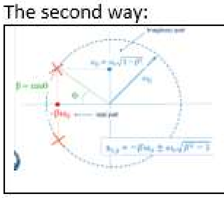
The first way:

$$s^2 + 2\beta\omega_0 s + {\omega_0}^2$$

$$2\beta\omega_0 = 2$$

$$\beta = \frac{2}{2\omega_0} = \frac{1}{\sqrt{3}}$$

The second way:



$$s^2 + 2s + 3$$

Has poles at $p1 - 1 + \sqrt{2}j$ *and* $p2 - 1 - \sqrt{2}j$

$$\theta = \tan^{-1}\left(\frac{\sqrt{2}}{1}\right) = 54.7°$$

$$\beta = \cos 54.7 = 0.578$$

$$M = e^{\frac{-\beta\pi}{\sqrt{1-\beta^2}}} = e^{-2.2} = 0.11$$

The overshoot that corresponds to $\beta = \frac{1}{\sqrt{3}}$ is 11%

Grading instructions:

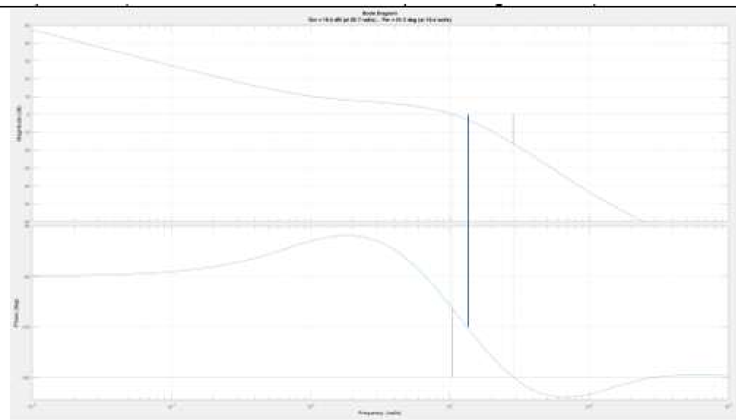
10 points: fully correct solution incl. correct explanations.

Deduct 2 points for minor calculation errors (both a and b)

For a) any other method to find the correct answer is also correct, if correctly done.

Problem 6

Answer 6



a) 8 pts

a) The gain margin is found at the frequency where the phase plot crosses through -180°. This is at 28.7 rad/s. The gain at that frequency is -16.4dB.
The gain margin GM=0 - -16.4dB = **16.4dB**

b) 7 pts

b) A phase margin PM = 45° is found where the phase plot crosses the -180°+45°=-135°. This happens at about 14 rad/s. The gain magnitude at that frequency is approximately -3dB. This means that we can increase the gain by 3dB before we meet the oscillation criteria.
Mag = 20 log(K) → K = 10^(3/20) = **1.41**

Grading instructions:
15 points: Correct answers with correct methods. Note that inaccuracies reading values from the graph are not counted as error, as long as indicated where values have been taken from.
Wrong frequency chosen and therefor wrong answer: deduct 2 points (both a and b)
No or wrong explanation with correct answer: deduct 2 points (both a and b)
For b) not converted magnitude shift to gain: deduct 2 points.

Problem 7

Nyquist stability criterium

Observe the Nyquist plot

$$Z_{RHP} = N_{CWE} + P_{OL\_RHP}$$

the closed-loop system is unstable if $Z > 0$

Z_{RHP} = Number of zeros in the Right Half Plane
N_{CWE} = Number of Clock Wise Encirclements of the point $-1 + j0$
P_{OL_RHP} = Number of poles of the Open Loop system in the Right Half Plane

If encirclements are in the counterclockwise direction, N_{CWE} is negative

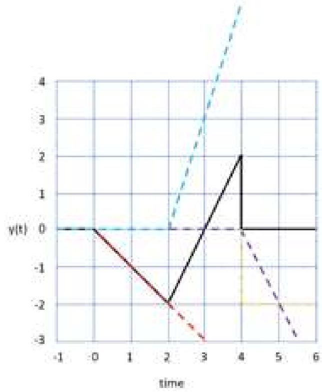
$N_{CWE} = -3$
$P_{OL\_RHP} = 3$

So $Z_{RHP} = 0$
This means the system is stable.

Grading instructions:
10 points: fully correct solution incl. correct explanation.
5 points: correct use of criterium $Z=N_{CWE} + P_{OL\_RHP}$ but incorrectly counted the counterclockwise encirclements.
< 5 points: everything else, at the discretion of the examiner.

Problem 8

Solution is found by decomposing the function y(t) in $y(t)$+ $y(t)$+ $y(t)$+ $y(t)$



$$y(t)\begin{cases} 0, & t<0 \\ -t, & t\geq 0 \end{cases} \xrightarrow{\mathcal{L}} -\frac{1}{s^2}$$

$$y(t)\begin{cases} 0, & (t-2)<0 \\ 3(t-2), & (t-2)\geq 0 \end{cases} \xrightarrow{\mathcal{L}} +\frac{3}{s^2}e^{-2s}$$

$$y(t)\begin{cases} 0, & (t-4)<0 \\ -2(t-4), & (t-4)\geq 0 \end{cases} \xrightarrow{\mathcal{L}} -\frac{2}{s^2}e^{-4s}$$

$$y(t)\begin{cases} 0, & (t-4)<0 \\ -2, & (t-4)\geq 0 \end{cases} \xrightarrow{\mathcal{L}} -\frac{2}{s}e^{-4s}$$

$$y(t) \xrightarrow{\mathcal{L}} \mathrm{Y(S)} = \xrightarrow{\mathcal{L}} \frac{1}{s}\left(-\frac{1}{s}+\frac{3}{s}e^{-2s}-\frac{2}{s}e^{-4s}-2e^{-4s}\right)$$

Grading instructions:
10 points: fully correct solution incl. correct explanation of separate parts (either in words or in drawing).
6 points: incorrect use of time shift in otherwise correct solution.
5 points: incorrect transform to Laplace domain.

Problem 9

The **P-action** increases the loopgain of the control system. The larger the loopgain the faster and more accurate the system is because loopgain reduces static deviation. Too much loopgain leads to overshoot and possibly even instability.

The **I - action** (integrator) will place a pole in the origin. This removes the static deviation because it makes the loopgain infinite for ω=0 (t= ∞)

The **D – action** (differentiator) will place a zero in the origin. This will generate error signal based on the derivative of the input signal. During the step input the derivative is large which helps to make the step response faster.

Grading instructions:
5 points: fully correct solution.

3 points: If only two actions are described correctly

1 point if only one action is described correctly