I STAGE ONE EXERCISES

Problem 1

An engineer is trying to control a electric machine with the given electric circuit in Fig. I.1. The induction coil L_x is wrapped on the stator's electromagnet to generate magnetic field to rotate the rotor. The electrician is using an current source $i(t)$ to provide electricity to the circuit and the electrician to should control the electric current $i_x(t)$ that flow through the inductor L_x such that the motor can run perfectly. Assume perfect current source, resistors, capacitor and inductor.

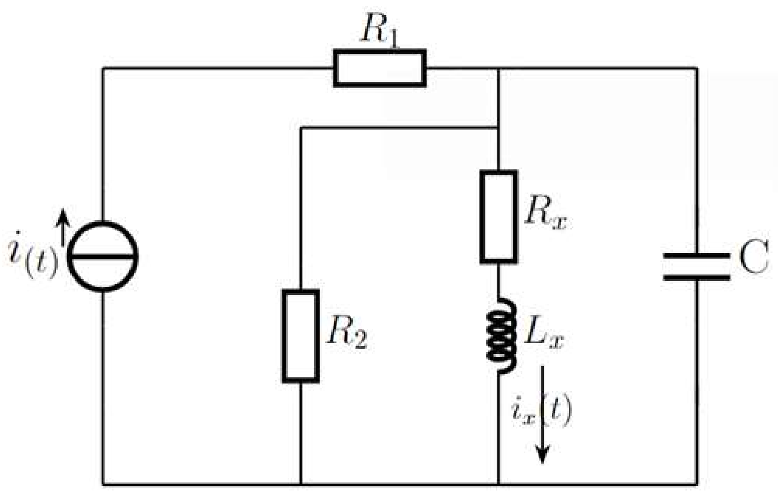

Figure I.1: The electric circuit designed by the motor electrician.

Carefully inspect and analyze the circuit, then answer the following questions:

1. Derive the transfer function $H(s) = \frac{I_x(s)}{I(s)}$. The numerator and denominator should all be polynomials about s with non-negative exponents!

2. Given the following values of the components in the circuit:
   $R_1 = 50\,\Omega, R_x = 1\,\Omega, R_2 = 40\,\Omega, L_x = 5\,H, C = 6.636\,F$,
   answer the following problem:
   Indicate the poles and zeros in this transfer function you have derived. If the system do not have poles and/or zeros, please also mention in your answer that there is no pole and/or zero.

Problem 2

Consider a Proportional-Integral-Derivative (PID) controller employed in a unit feedback closed loop system.
Explain: why you should never use the derivative controller alone?
Motivate your answer in less than 5 sentences and/or 3 equations.
Hint: Draw the control loop if you think it's helpful to assist your reasoning.

Problem 3

Describe the Ziegler-Nichols tuning method for PID controllers, and briefly motivate why this method can be effective.

Problem 4

Assume zero initial condition, given an ordinary differential equation:

$$\frac{d^4 x(t)}{dt^4} - 3\frac{d^2 x(t)}{dt^2} - 54x(t) = \frac{d^2 x(t)}{dt^2} + \frac{dx(t)}{dt}.$$

The input signal is $x(t)$ and the output signal is $x(t)$. Answer the following questions:

1. Find the transfer function of this system corresponding to the given ODE.
2. Is this system time-invariant? Why?
3. Is this system causal? Why?
4. Find the poles and zeros of this system. Visualize them in the s-plane.
5. Is this system stable? Why?

Problem 5

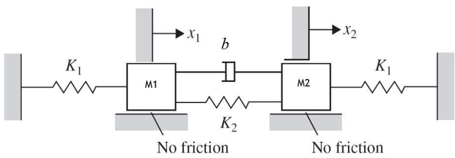

Figure I.2: The translational mechanical system.

There are two masses M1 , M2 are connected by springs and viscous dampers. An external force $f(t)$ acts on M1. The model is shown in Fig. I.2. Anwer the following quesitons:

1. Draw the free body diagrams of M1 and M2.
2. Find the differential equations based on the free body diagram that describes the relation between the displacement $x_2(t)$ of M2 and the external force $f(t)$.
3. Assume the springs are identical, and the masses are of the same weight: $m = M1 = M2, k = K1 = K2$. Find the transfer function $G(s) = \frac{X_2(s)}{F(s)}$.

Problem 6

A periodic function $x(t)$ defined in $t \in (0, \infty)$ is shown in Fig. I.3.

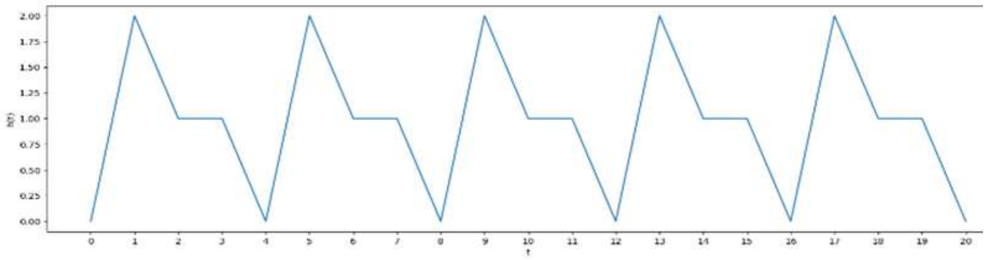

Figure I.3: The periodic function.

Answer the following questions: (You must show the intermediate steps!)

1. What is the period T of $x(t)$?
2. Derive the time domain representation of $x(t)$.
3. Find the s-domain representation $Y(s)$ using the Laplace transform.

Problem 7

Find the inverse Laplace transform of the following functions:

1. $F(s) = \frac{s+1}{s^2(s+4)}$
2. $F(s) = \frac{s+1}{(s+3)(s^2+s+5)}$
3. *Challenging:* $F(s) = \frac{s(1+e^{-s}+e^{-2s})+e^{-3s}}{s(s+1)}$

Problem 8

In the modern era, greenhouses are becoming increasingly advanced with the integration of automated control systems. As a mechatronic designer, your task is to develop a control system that maintains a stable CO2 concentration in the greenhouse, ensuring improved farming yields.

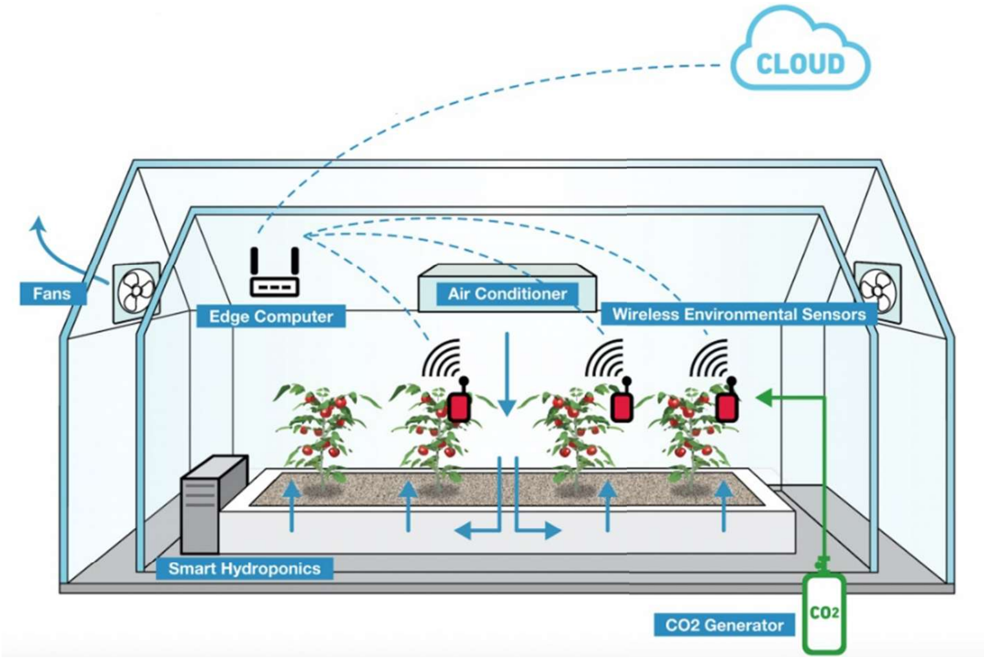

Figure I.4: Smart greenhouse system illustrations.
(source: https://www.cassianetworks.com/blog/iotforsmartfarming/)

1. Describe in words how the CO2 concentration control system works. Use in your description all of the following keywords:

   1. objective of the control system,
   2. reference variable(s),
   3. controlled variable(s),

   1. sensor(s),
   2. controller(s),
   3. feedback loop(s),
   4. 2 examples of possible disturbances

   If it helps your description please illustrate with a sketch or drawing. Use between 70-140 words for your description (max. half page handwritten)

2. Draw a functional block diagram of the CO2 concentration control system as described in a). Indicate input and output signals, intermediate signals and main subsystems. Use system-specific functional names for blocks and signals.

Problem 9

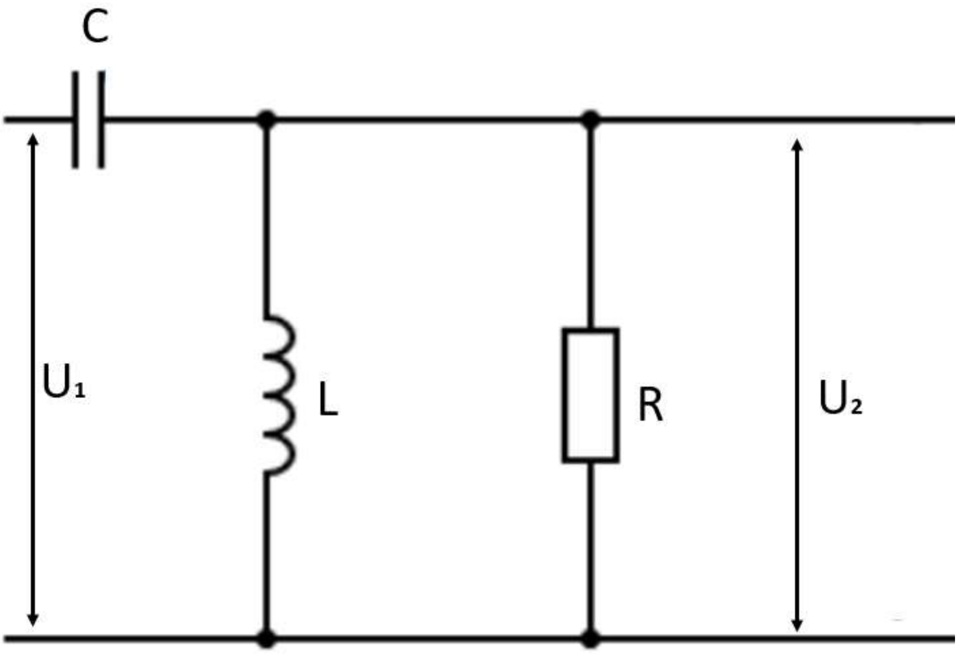

Figure I.5: The RLC Circuit.

Given the circuit in Fig. I.5, U_1 is the supplied voltage, and U_2 is the output voltage.

1. Find the differential equation that describes the relationship between input and output.

2. Find the transfer function.

3. Draw the block diagram related to this circuit. And demonstrate how you can simplify this block diagram to find the transfer function you have derived above. Mark the signals on the circuit and the block diagram.

Problem 10

You are working in a factory, responsible for a specific process control problem. However, you do not know how accurate your controlled process should be. Your technical manager tells you that after 3 time constants, then you can regard the process is settled.

Answer the following questions:

1. What is a time constant? How large is one time constant?

2. What is the error tolerance of this controlled process?

3. If I have another controlled process that requires precise controll with an error tolerance lower than 0.5%, how many time constants does that requirement correspond to?