J STAGE TWO EXERCISES

Problem 1

We have two sets of Bode plots in Fig. J.1 for two functions:

$$f(t) = \frac{1}{2}\sin\omega_1 t \quad g(t) = \cos\omega_2 t$$

Answer the following questions:

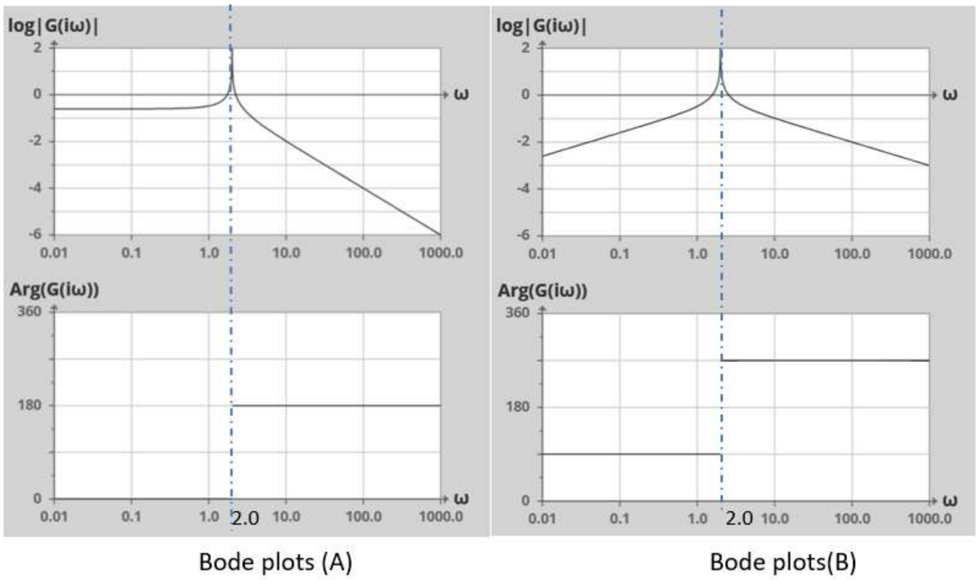

Figure J.1: Two sets of Bode plots.

1. What are the Laplace transforms F(s) from f(t) and G(s) from g(t)?
2. Match the Bode plots with the mappings(functions) $F(s = j\omega)\&G(s = j\omega)$. Sufficiently motivate why. Use math equations or numerical examples when necessary.
3. Draw the Nyquist plot of $F(s = j\omega)$ and $G(s = j\omega)$.

Problem 2

Read the following statements about open-loop transfer functions carefully.

1. An arbitrary open-loop transfer function H(s) has no zeros and 2 poles. The real parts of the poles of H(s) are negative. We employ a proportional controller K to H(s) and establish an unit-feedback closed loop control system.
   As K increase from zero to infinity, the closed-loop system will always be stable.

2. An arbitrary open-loop transfer function H(s) has no zeros and 4 poles. The real parts of the poles of H(s) are negative. We employ a proportional controller K to H(s) and establish an unit-feedback closed loop control system.
   As K increase from zero to infinity, the closed-loop system will always be stable.

3. An arbitrary open-loop transfer function H(s) has at least one zero in the right-half complex plane in s-space. We employ a proportional controller K to H(s) and establish an unit-feedback closed loop control system.
   If all poles of H(s) are in the left-half complex plane, the closed-loop system will always be stable as K increase from zero to infinity.

Determine whether the statements are **TRUE** or **FALSE**, and explain why. You must provide at least 1 example per statement using Root Locus plot to support your answer.

Problem 3

Given the Bode plots in Fig. J.2, an open loop process with transfer function $G(s)$. The process is placed in a normal feedback loop (negative feedback and unity feedback) and a controller is used with a gain K (with $K > 0$).
Indicate whether the following statements are **TRUE** or **FALSE**. Motivate your answer.

1. The system is stable.
2. If I increase the gain to a value sufficiently large, the system will become unstable.

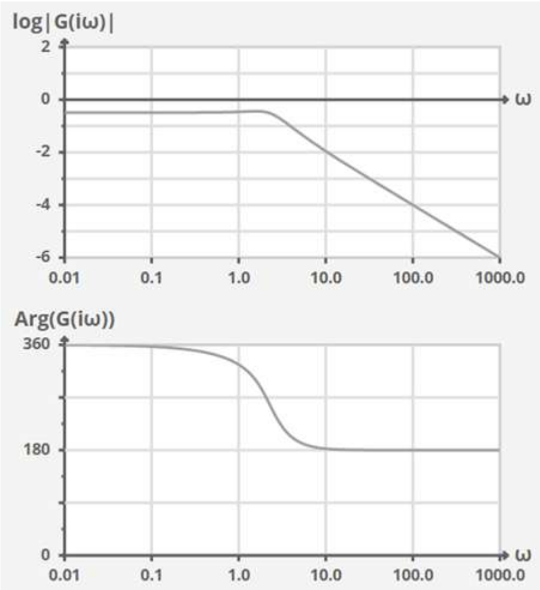

Figure J.2: The Bode plots.

Problem 4

Given the Bode plots in Fig. J.3, find the:

1. Gain Margin
2. Phase Margin
3. Stability of the system

You need to draw on the plots where did you find the gain margin & phase margin. Explain your steps.

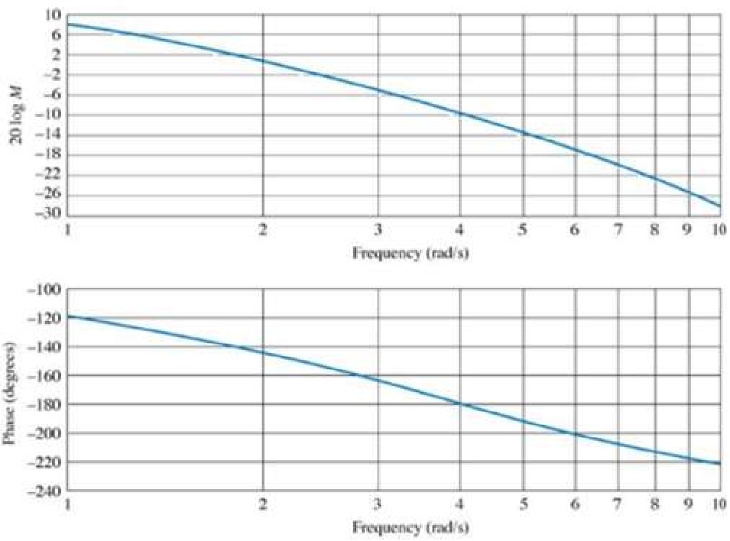

Figure J.3: The Bode plots.

Problem 5

Given the Root Locus plots of 3 systems (A), (B), and (C) in Fig. J.4. The DC-gain of both systems are 3. Determine the differential equations in the time domain for systems (A) and (B). Explain how you determined the differential equation. (Poles are marked with an 'x' and the zeros are marked with a circle 'o')

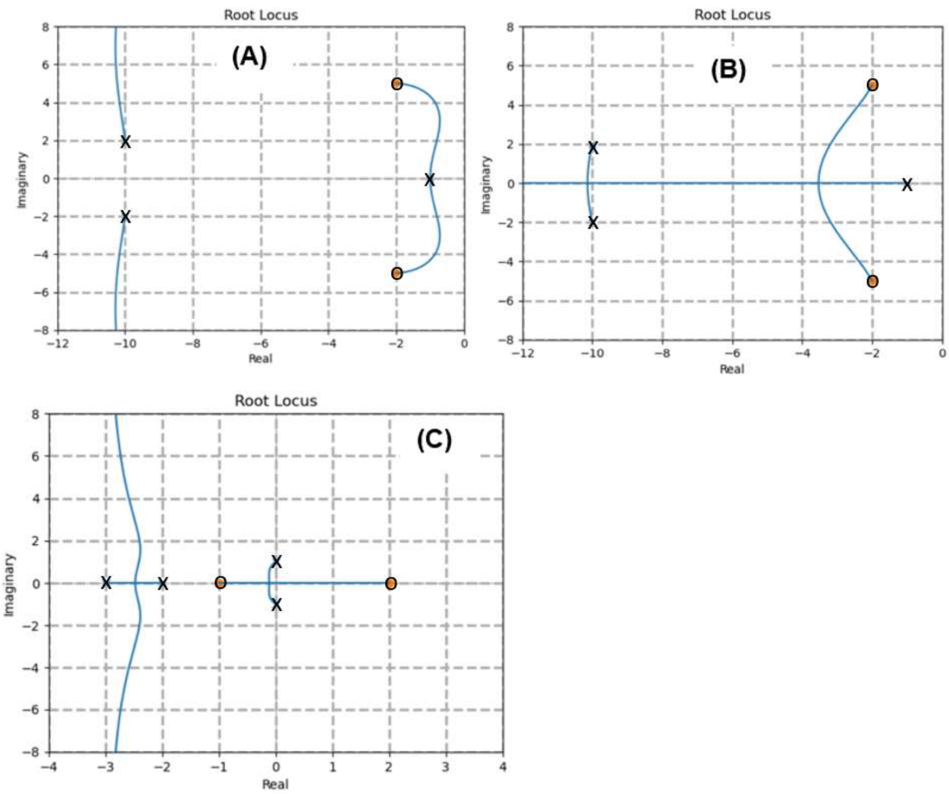

Figure J.4: The Root Locus plots.

Problem 6

Consider a system with a proportional controller K connected inseries with a process $H(s)$. The controller and the process is placed in a unit negative feedback control loop. The open loop transfer function:

$$k \cdot H(s) = \frac{K(s+4)}{s^2 - 2s + 10}$$

Answer the following questions:

1. Sketch the root locus.
2. Indicate in your root locus sketch where the system is stable and where the system is unstable.
3. Find the range of values of the controller gain K for which the system is stable.

Problem 7

The following Nyquist plots (A), (B), (C) in Fig. J.5 are obtained for the same process $L(s)$ with different P-controller parameters K_p. Assume the closed loop system is unity feedback.

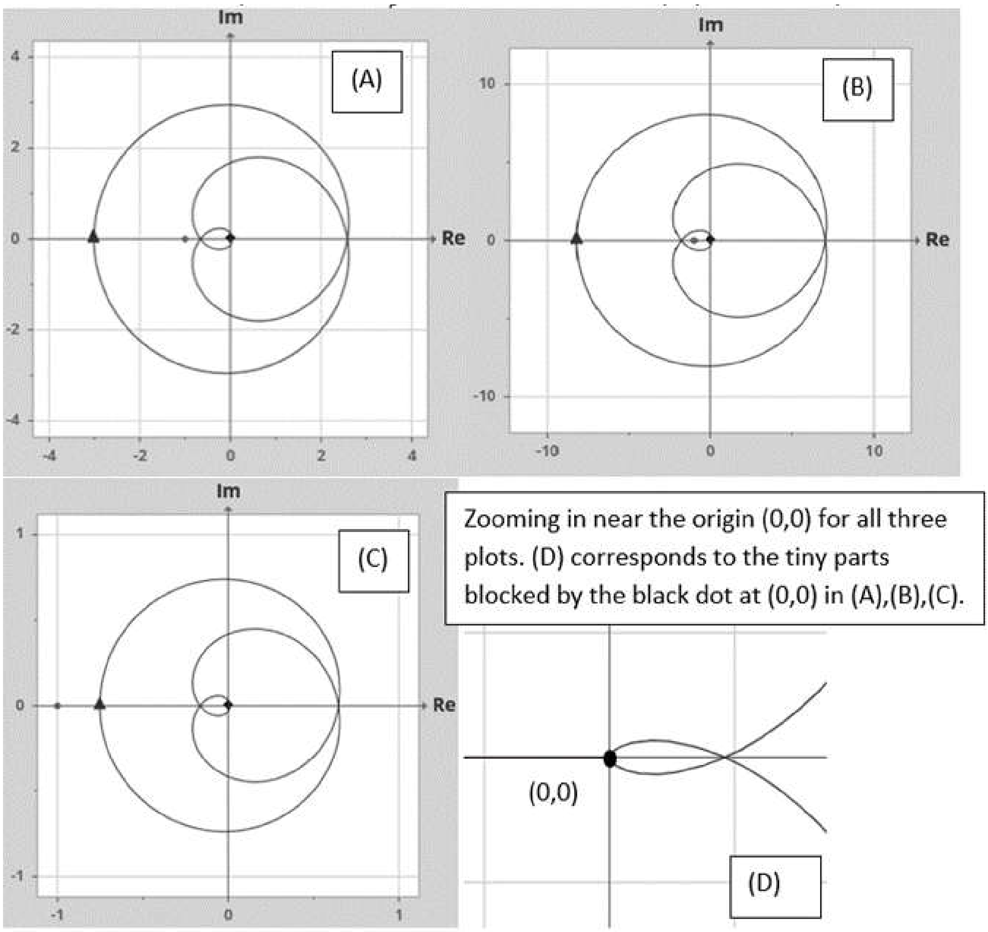

Figure J.5: The three Nyquist plots.

1. We know that K_p values are 0.5; 2.2; 6. Match the K_p values with the plots.
   Nyquist plot (A) K_p =? Nyquist plot (B) K_p =? Nyquist plot (C) K_p =?
2. Assume L(s) have 1 pole in the right half complex plane.
   Find out the stability of the closed loop system for (A), (B), and (C) using Nyquist criterion

Problem 8

Now you are a control systems engineer working in a company. Your client states that they desire a system with the fastest response possible but with the least possible overshoot. Because the system needs to work in critical environment, the systems can tolerate some overshoot but must not be out of control as the proportional controller K varies. You have 3 systems in your company to choose from, their complex poles and zeros are given in Table 1 below.

Table 1 The list of available systems

Systems	Poles	Zeros
System 1	s = -2 and s = -7	No zeros
System 2	s = -3+10i and s = -3-10i	No finite zeros
System 3	s = -3	s = 3

Table 1: The three Nyquist plots.

1. Sketch the root locus plots of these 3 systems.
2. Without making any explicit calculations, give arguments about which system(s) you would like to choose to provide to the clients. Give short arguments about why you choose or not to choose for each system.(1 or 2 sentences per system)

Problem 9

Sketch the Bode plots for the following systems:

1. $\frac{s+1000}{s+10}$
2. $\frac{(s+1)(s-1)}{s^2+10}$
3. $\frac{s+1}{s^2+s+21}$
4. *Challenging:* $10\frac{(s^2+s+25)(s-1)}{s^3+100s^2}$
5. *Challenging:* $e^{-0.1s}\frac{s-0.1}{s+10}$

Problem 10

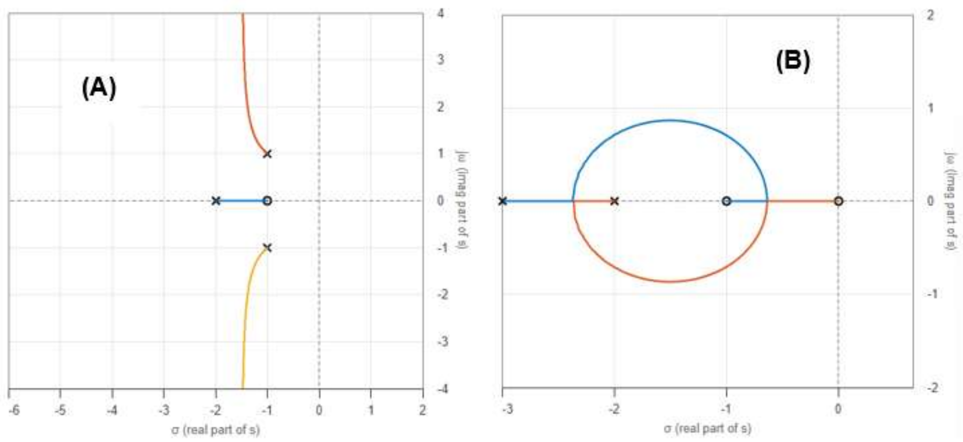

Figure J.6: The 2 root-locus plots (A) and (B).

There are 2 root locus plots of 2 systems in Fig. J.6.

1. What are the transfer functions corresponding to the root locus plots? Are they stable?

2. Find the maximally allowable K for these transfer functions to be placed in a standard unit negative feedback control loop with a simple proportional controller K.

For the 2 systems in Fig. J.6, each systems are joined by a new process $(s-3)$ in series.

3. Find the new transfer function.

4. Draw the new root locus for the new system.

5. Find the maximally allowable K for these new systems to be placed in a standard unit negative feedback control loop with the simple proportional controller K.

6. Can you comment something else about the difference between old and new systems? (At least one point for each.)