K TEST EXAM ONE

Introduction & Instructions

This is the regular exam of BCS 2024 FALL.

- Duration: **3 hours**
- Number of problems: **9**
- Allowed material: **Writing gear, calculator**
- Calculation of final scores: $\frac{\textbf{score\_obtained}}{10}$, round to 1 decimal
- Note for grading: correct solutions without reasoning do **not** grant points.

Problem 1 (15 points)

Given the block diagram in Fig. K.1.

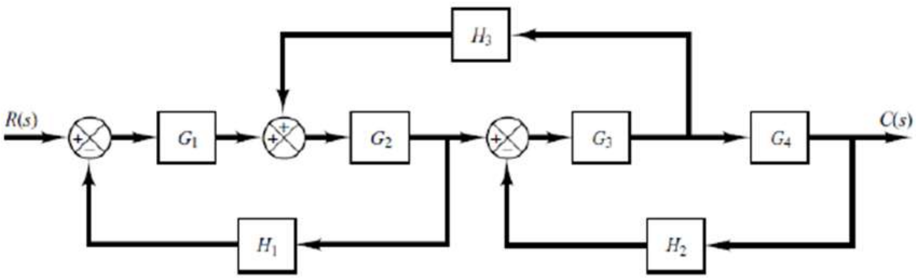


Figure K.1: Block diagram

Find the transfer function:

$$H(s) = \frac{C(s)}{R(s)} .$$

Show at least two intermediate steps used to find the solution.

Problem 2 (10 points)

Given the following transfer function $H(s)$.

$$H(s) = \frac{5 \cdot 10^4 s}{s^2 + 505s + 2500}$$

Sketch the Bode plots of $H(s)$, including both magnitude plot and phase plot.
Please clearly indicate the asymptotes.
Sketch the Bode plots on the corresponding sketch sheet and remember to also submit the sketch sheets!
Explain how you obtained your sketch (show intermediate steps like asymptotes, how you find cornor frequencies, etc.) on your answer sheet.

Sketch sheet problem 2

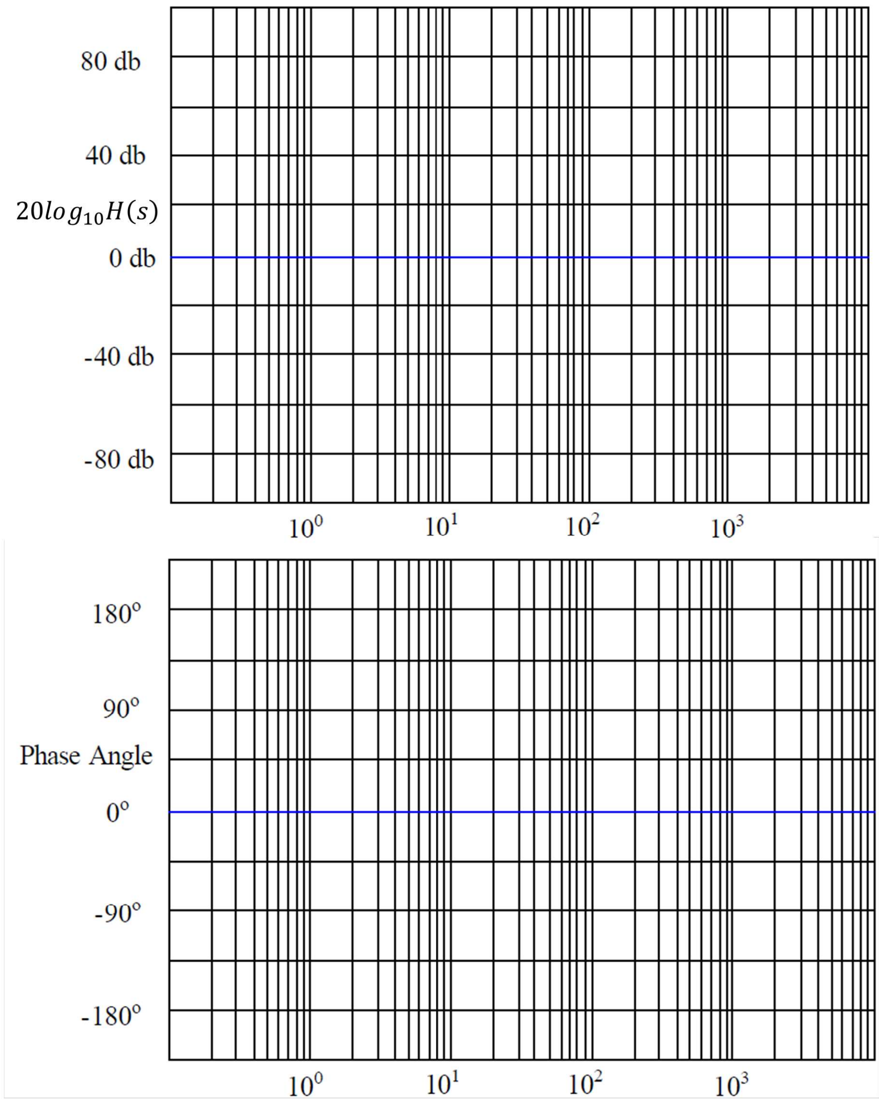


Figure K.2: Bode plots sketch sheet

Problem 3 (15 points)

Given the Bode plot of an open loop process with transfer function $G(s)$ in Fig. K.3. The process is placed in a standard unity negative feedback loop and a controller is used with a gain $K = 1$ (with $K > 0$).

Find and clearly indicate the gain margin (GM) in dB and phase margin (PM) in degrees on the sketch sheet.

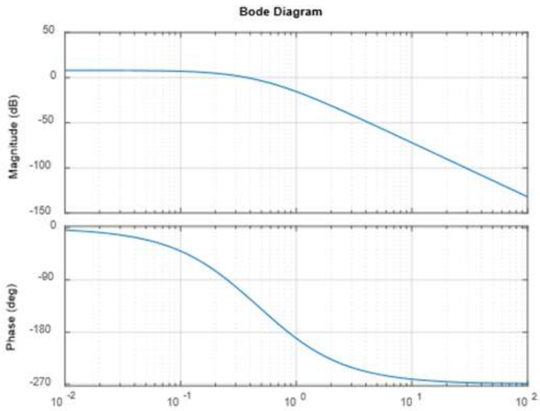


Figure K.3: Bode plots of the transfer function $G(s)$.

Determine whether the following statements are TRUE or FALSE:

1. The system is stable.
2. The DC gain of $G(s)$ is 0 dB.
3. When the controller gain $K = GM + 10\,\text{dB}$, the system will remain stable.

Fully motivate your judgement.

Sketch sheet problem 3

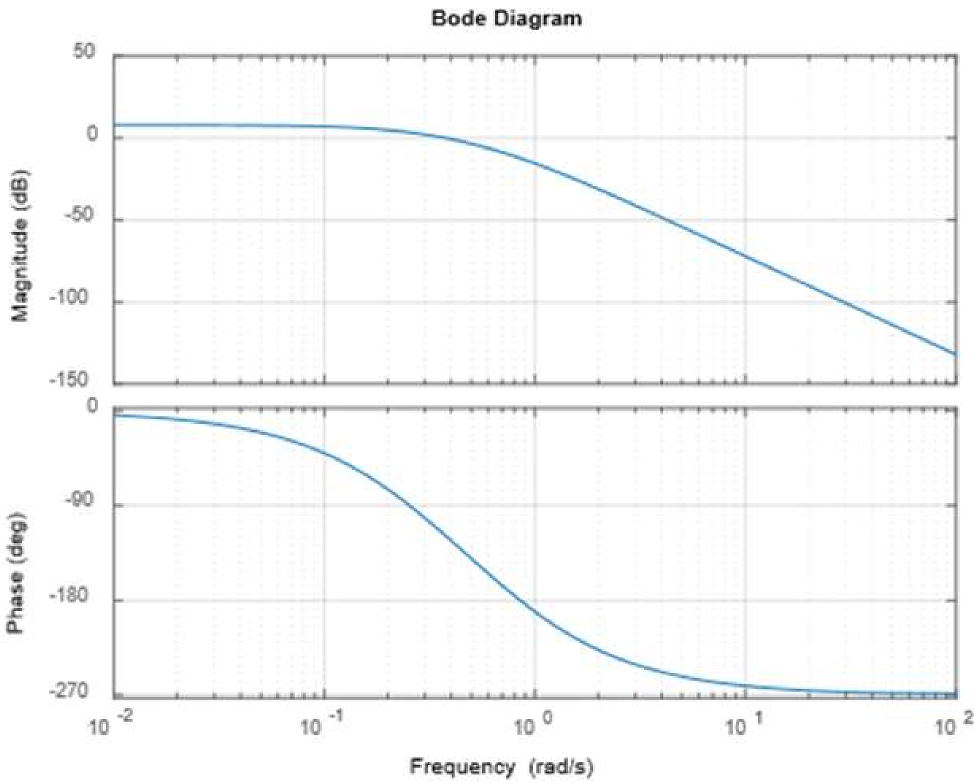
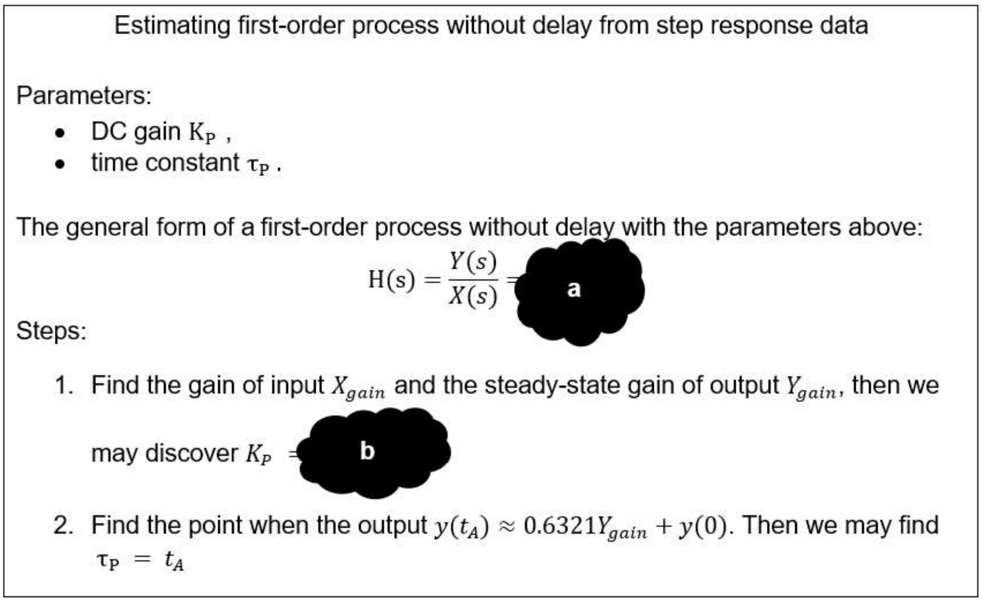


Figure K.4: Bode plots of the transfer function $G(s)$.

Problem 4 (10 points)

There is one unknown process H whose transfer function is denoted by $H(s)$. We do not know the exact transfer function of H(s), but we may conduct experiment in the time domain to estimate the transfer function. We assume that this process H is a *first-order (rising) process without delay*.

After obtaining the unit step response data of H, the engineer recorded the steps he did to estimate the transfer function. Unfortunately, his note book was destroyed accidently by some paint. Answer the following questions:


Estimating first-order process without delay from step response data

Parameters:
- DC gain K_P ,
- time constant τ_P .

The general form of a first-order process without delay with the parameters above:

$$H(s) = \frac{Y(s)}{X(s)} = \text{a}$$

Steps:

1. Find the gain of input X_{gain} and the steady-state gain of output Y_{gain}, then we may discover $K_P =$ b
2. Find the point when the output $y(t_A) \approx 0.6321 Y_{gain} + y(0)$. Then we may find $\tau_P = t_A$

Figure K.5: The engineer's partially destoryed notebook.

1. Fill in the 'a' spot on the engineer's notebook.
2. Fill in the 'b' spot on the engineer's notebook.
3. Explain the origin of the 'magical number' 0.6321 in step 2.

Problem 5 (10 points)

The input $x(t)$ to system $h(t)$ is an impulse function. The transfer function in s-domain $H(s)$ is described by the following equation:

$$H(s) = \frac{s + 10}{s^2 + 20s + 101}$$

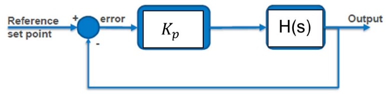
Find the impulse response of the output $x(t)$.

Problem 6 (10 points)

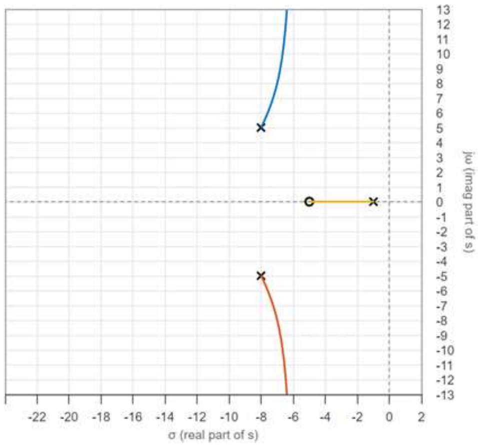
Fig. K.6 depicts an open-loop system H(s) in a negative unit feedback loop with a controller K and the root-locus plot of H(s) in the complex s-plane. The DC-gain of the open-loop process H(s) is 5. Find the solutions to the following problems:

1. FInd the open-loop transfer function H(s).

2. Find the closed-loop transfer function with controller gain $K_p = \frac{1}{89}$.



(a) The control loop.



(b) The root locus plot.

Figure K.6: The control system and root locus analysis.

Problem 7 (10 points)

There is an open-loop process with a transfer function H(s) that have 3 poles:

$$s = -2 \quad s = -1 + 1.99j \quad s = -1 - 1.99j$$

Fig. K.7 demonstrates the Nyquist plot of H(s). Assuming a proportional controller K deployed to control the open-loop process with unity negative feedback loop in the closed loop system. Find the solutions to the following problems:

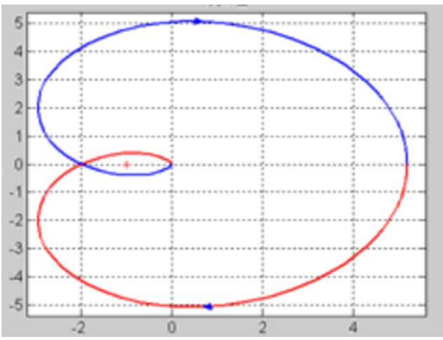


Figure K.7: The Nyquist plot for the transfer function H(s).

1. Determine the stability of the closed-loop system using Nyquist stability criteria.

2. What should we do to the proportional controller K to make the system stable? Analyze using the property of the Nyquist plot, do **not** use explicit calculation.

Problem 8 (10 points)

Draw the block diagram of a PID controller with the parallel structure and derive the transfer function of the PID controller.

You only need to work on the controller! Not the control loop!

Problem 9 (10 points)

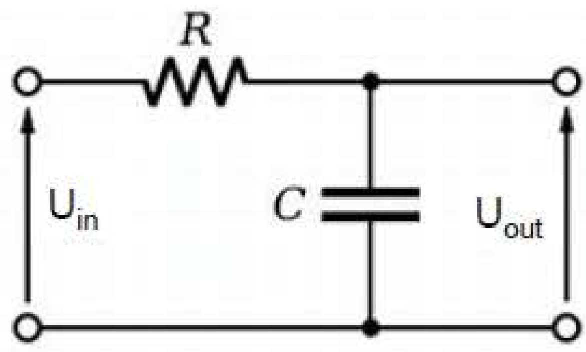


Figure K.8: First order RC low-pass filter.

An example first order RC low-pass filter is shown in Fig. K.8. Find the solutions to the following questions:

1. Find the transfer function of the RC low-pass filter.
2. What is the bandwidth of this RC low-pass filter?
3. If we want the bandwidth of this RC low-pass filter to be 1kHZ, and the resistance of the resistor R=1kΩ. How large is the capacitance C?